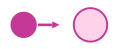


AP Physics 1

The exam: Wednesday, May 5, 2027, 3 hours of testing

The eight units, and each one's share of the multiple-choice questions

Unit 1 Kinematics 10–15% 	Unit 2 Force and translational dynamics 18–23% 	Unit 3 Work, energy, and power 18–23% 	Unit 4 Linear momentum 10–15% 	Unit 5 Torque and rotational dynamics 10–15% 	Unit 6 Energy and momentum of rotating systems 5–8% 	Unit 7 Oscillations 5–8% 	Unit 8 Fluids 10–15% 
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Free-response questions can draw on any unit, and many combine two or three. Since fall 2024 the course has included fluids and has left out circuits, waves and sound, so skip those chapters in older review books.

Section I: multiple choice, 50% of your score

Section II: free response, 50% of your score

Multiple choice

42 questions in 85 minutes, about 2 minutes a question

Question 1

10 points
20–25 min

Question 2

12 points
25–30 min

Question 3

10 points
25–30 min

Question 4

8 points
15–20 min

0 min

85 min

180 min = 3 hours

The four free-response questions, always in this order

- Mathematical Routines** 10 points
You derive equations in symbols, calculate numbers with units, and make a claim that you back up with physics. Expect a diagram or a graph along the way, such as a free-body diagram.
- Translation Between Representations** 12 points
You show one situation in several ways: a diagram (forces, energy bars or momentum arrows), an equation you derive and a graph you sketch. Then you explain why two of your answers agree, or use them to predict a change.
- Experimental Design and Analysis** 10 points
First you write a procedure with ordinary lab equipment: you change one quantity, measure how another responds, and repeat each trial. Then you plot data you are given, draw a best-fit line and turn its slope or intercept into the answer.
- Qualitative/Quantitative Translation** 8 points
You make a claim and justify it in words, derive an equation for the same situation, then show that it supports your claim.

The suggested times are a guide; only the 95-minute total counts.

How it's scored

- Multiple choice counts right answers only. A wrong answer costs nothing, so answer all 42.
- Free-response points are awarded for separate pieces of a solution: the starting equation, the substitution, the answer with its unit, a reason. A bare final answer can lose most of them.
- "Justify" means physics in words: name the forces or energies involved and say why they lead to your answer.

The reference sheet

- You get it on paper and in Bluebook. It has the formulas for all eight units, with no notes on when each one applies.
- It gives $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ and $1 \text{ atm} = 1.0 \times 10^5 \text{ Pa}$. It lists $g = 9.8 \text{ m/s}^2$, but exam questions are written for $g = 10 \text{ m/s}^2$; 9.8 or 9.81 used correctly also earns full credit. This sheet uses 10.
- It lists metric prefixes, geometry formulas, and sin, cos and tan of 0° , 30° , 37° , 45° , 53° , 60° and 90° . The 37° and 53° values come from a 3-4-5 triangle, which many exam questions use.
- Unless a question says otherwise, assume no air resistance, ideal springs, strings and fluids, pipes full of fluid, and a frame that isn't accelerating.

On exam day

- You read every question and answer the multiple choice in the Bluebook app. You write free-response answers by hand in a paper booklet, and only work in the booklet is scored.
- A calculator is allowed on both sections: four-function, scientific, or a graphing model from College Board's approved list. Bluebook also has Desmos scientific and graphing calculators built in.
- Bring a ruler or straightedge for best-fit lines.
- These timings are new for May 2027. The 2025–26 format had 40 multiple-choice questions in 80 minutes and 100 minutes of free response.

Set the angle mode before you start

Use degrees for angles like 37° , and radians for $x = A \cos(2\pi ft)$. In degree mode, $\sin 30$ gives 0.5.

TI-84 Plus CE. Press **mode**, arrow down to the row RADIAN DEGREE, highlight the one you want, press **enter**, then **2nd mode** to quit.

TI-Nspire CX. Press **on** for the home screen, then 5 (Settings) and 2 (Document Settings). Set Angle to Degree or Radian, select Make Default, press **enter**, then confirm with OK.

Desmos. In Graphing, open the wrench and choose Degrees or Radians. In Scientific, toggle DEG/RAD on the keypad.

Unit 1 Kinematics

10–15%
of multiple choice

Kinematics describes how things move without asking why. You work with position, velocity and acceleration as signed numbers, as graphs and as equations, first along a line and then for projectiles. Every later unit uses these tools, and free-response questions often include a kinematics step, such as sketching a projectile's velocity graphs or finding a launch speed from the height it reaches.

Not on the exam: calculations with changing acceleration (you only sketch and describe it), and relative velocity in two dimensions, such as a boat crossing a river.

1.1 Scalars and vectors in one dimension

A scalar is just a size: distance, speed, time, mass. A vector has a size and a direction: displacement, velocity, acceleration, force. Along a line, choose which way is positive once, and let the sign carry the direction. Displacement is the change in position and can be negative; distance is the length of the path and is never negative.

Worked example

You walk 30 m east, then 10 m back west, in 8.0 s. East is positive. Find the displacement, the distance, the average velocity and the average speed.

- 1 Displacement: $+30 - 10 = +20$ m. Distance: $30 + 10 = 40$ m.
- 2 Average velocity = $+20/8.0$; average speed = $40/8.0$.

Average velocity +2.5 m/s, average speed 5.0 m/s: velocity uses displacement and speed uses distance.

Common mistake

Reading a minus sign as "slowing down." A velocity of -5 m/s means moving at 5 m/s in the negative direction. It says nothing about whether the object is speeding up. And speed is never negative: write "velocity = -5 m/s," not "speed = -5 m/s."

1.2 Displacement, velocity, and acceleration

Velocity is the rate at which position changes, and acceleration is the rate at which velocity changes. An average uses the change over a whole interval. Instantaneous velocity is the slope of the tangent line on the position graph; instantaneous acceleration is the slope of the tangent line on the velocity graph. Whether an object speeds up depends on two signs: when velocity and acceleration have the same sign it speeds up, and when they have opposite signs it slows down. A cart moving in the negative direction while slowing down has $v < 0$ and $a > 0$.

Not on the reference sheet $\Delta x = x_f - x_i$ $v_{\text{avg}} = \Delta x / \Delta t$

$a_{\text{avg}} = \Delta v / \Delta t$ average speed = distance / Δt

Common mistake

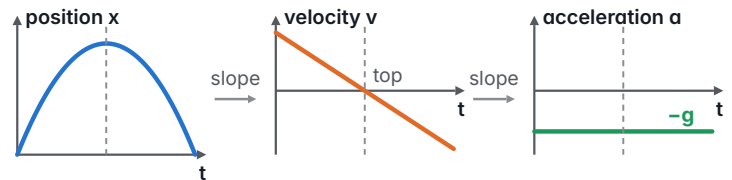
Subtracting speeds to find the change in velocity of a bounce. A ball hits the floor at 6.0 m/s and leaves at 4.0 m/s, so it is tempting to say its velocity changed by 2.0 m/s. But the velocity reversed. With up as positive, $\Delta v = (+4.0) - (-6.0) = +10.0$ m/s. Over 0.050 s of contact, that is an average acceleration of 2.0×10^2 m/s², upward.

Common mistake

Taking $\Delta x / \Delta t$ as the final velocity. It is the average. A cart that starts from rest and speeds up steadily for 2.0 m in 2.0 s averages 1.0 m/s, but its final speed is 2.0 m/s, twice the average. Graders see lab answers that are off by exactly this factor of two.

1.3 Representing motion: graphs

One motion can be shown as a motion diagram, as graphs of position, velocity and acceleration against time, or as equations. Free-response questions often give you one and ask for another, and they all have to agree at every instant.



The ball is thrown straight up and caught, with up as positive. At the top $v = 0$, but a is still -10 m/s²: it never changes during the flight.

Graph	Its slope is the	The area under it is the
position vs time	velocity	(not used)
velocity vs time	acceleration	displacement
acceleration vs time	(not used)	change in velocity

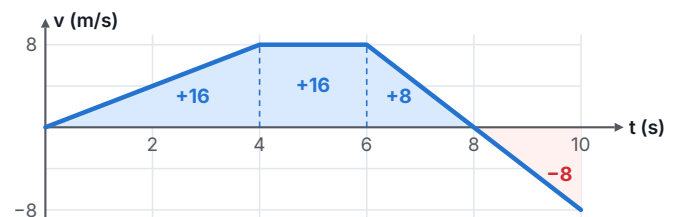
On a position graph, steeper means faster, flat means at rest, and a peak or a valley is a turnaround where $v = 0$. A position graph that bends upward like a cup (concave up) has positive acceleration; one that bends downward has negative acceleration. Where two position graphs cross, the objects are at the same place at that instant; their speeds can still differ.

Common mistake

Assuming that slowing down means negative acceleration. Signs depend on which way you call positive. Say downhill is positive and a cart rolls up a ramp: it moves uphill, so its velocity is negative. Gravity pulls it downhill the whole time, so its acceleration is positive even while it slows down. Its position graph slopes down and curves upward.

Worked example

A cart starts at $x = 0$ with the velocity graph below. Find its displacement and the distance it travels in 10 s, and when it is farthest from the start.



- 1 Displacement adds the areas: $16 + 16 + 8 - 8 = 32$ m. Distance adds them all as positive: 48 m.
- 2 The velocity changes sign at 8 s, so the cart turns around there, $16 + 16 + 8 = 40$ m from the start.

Displacement +32 m, distance 48 m, farthest at 8 s.

1.3 Representing motion with equations

When the acceleration is constant, three equations from the reference sheet link five quantities: Δx , v_0 , v , a and t . You usually know three and want a fourth: pick the equation that leaves out the one quantity you neither know nor need. The sheet writes them for the x direction; for vertical motion, use y . None of them works across a change in acceleration.

On the reference sheet $v_x = v_{x0} + a_x t$ $x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

Not on the reference sheet $\Delta x = \frac{1}{2}(v_{x0} + v_x) t$

(the area under a straight v - t line)

Worked example

A car at 12 m/s brakes at a steady 3.0 m/s² until it stops. How long does it take, and how far does it go?

- Forward is positive: $v_0 = 12$ m/s, $v = 0$, $a = -3.0$ m/s².
- $0 = 12 + (-3.0)t$, so $t = 4.0$ s.
- $0 = 12^2 + 2(-3.0)\Delta x$, so $\Delta x = 24$ m.

4.0 s and 24 m. At twice the speed, 24 m/s, the car needs four times the distance, 96 m, because stopping distance grows with the square of the speed.

Free fall. With up as positive, $a = -g = -10$ m/s² on the way up, at the top and on the way down. Without air resistance, the trip up takes as long as the trip down, and speeds match at equal heights.

Common mistake

Using one equation across a change in acceleration. A rocket launched vertically from rest that accelerates at 20 m/s² for 3.0 s and then coasts has two intervals. The first ends at 60 m/s and 90 m above launch; starting from 60 m/s with $a = -10$ m/s², it rises 180 m more, to 270 m. One equation for the whole flight can't give this.

In the lab question, turn a curve into a straight line. From rest, $x = \frac{1}{2}at^2$, so a graph of x against t^2 is a straight line with slope $a/2$. A best-fit slope of 0.40 m/s² gives $a = 2 \times 0.40 = 0.80$ m/s².

Common mistake

Blaming "human error." Name the cause and which way it pushes your answer: if you start the timer late, every time is too short, so your calculated acceleration is too large.

1.4 Reference frames and relative motion

Every velocity is measured by some observer. Write v_{AB} for the velocity of A as measured by B. Along a line, velocities add with their signs, and swapping the letters flips the sign. Observers moving at a steady velocity relative to each other measure different velocities but the same acceleration. A rider on a cart rolling at 2 m/s tosses a ball straight up: she sees it go straight up and down, someone on the ground sees a parabola, and both measure 10 m/s² downward.

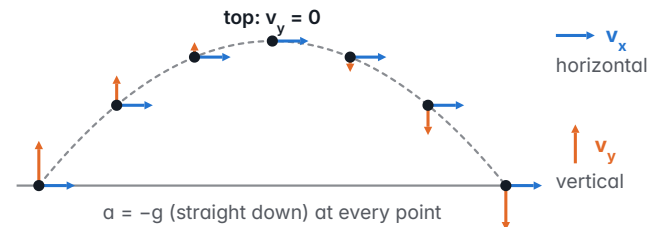
Not on the reference sheet $v_{AC} = v_{AB} + v_{BC}$ $v_{BA} = -v_{AB}$

Common mistake

Two cars drive toward each other at 25 m/s each, and students say their relative speed is zero because the speeds match. Call the westbound car W, the eastbound car E and the ground G, with east positive: $v_{WE} = v_{WG} + v_{GE} = (-25) + (-25) = -50$ m/s. They close the gap at 50 m/s.

1.5 Vectors and motion in two dimensions

Split a vector into perpendicular components. With the angle θ measured from the horizontal, $v_x = v \cos \theta$ and $v_y = v \sin \theta$; at 37°, a 20 m/s launch splits into 16 m/s across and 12 m/s up. A projectile is two straight-line motions that share one clock. Nothing pushes it sideways, so v_x stays constant. Vertically it is in free fall, with $a = -g$.



Not on the reference sheet $v_x = v \cos \theta$ $v_y = v \sin \theta$

$$v = \sqrt{v_x^2 + v_y^2}$$

Time in the air The vertical motion alone sets it, so a ball dropped and a ball launched sideways from the same height land together.

Range It is v_x times the time in the air.

Twice the launch speed At the same angle, it doubles the time in the air and makes the height and the range four times as large (equal launch and landing heights).

Worked example

A ball rolls off a 1.25 m high table at 3.0 m/s. How far from the table does it land, and how fast is it moving then?

- Vertically it starts with $v_y = 0$: $1.25 = \frac{1}{2}(10)t^2$, so $t = 0.50$ s.
- Horizontal, same t : $x = (3.0)(0.50) = 1.5$ m.
- At landing $v_y = (10)(0.50) = 5.0$ m/s down and $v_x = 3.0$ m/s, so the speed is $\sqrt{3.0^2 + 5.0^2} = 5.8$ m/s.

It lands 1.5 m from the table at 5.8 m/s.

Worked example

A ball leaves level ground at 20 m/s, 37° above the horizontal. Find its greatest height, its time in the air and its range.

- $v_x = 20 \cos 37^\circ = 16$ m/s and $v_{y0} = 20 \sin 37^\circ = 12$ m/s.
- At the top $v_y = 0$: $0 = 12^2 - 2(10)h$, so $h = 7.2$ m, reached after $12/10 = 1.2$ s.
- It lands at its launch height after $2(1.2) = 2.4$ s, so the range is $(16)(2.4)$.

7.2 m high, 2.4 s in the air, 38 m away.

Worked derivation

A water jet leaves at angle θ and rises a height h . Find its launch speed. Start from $v_y^2 = v_{y0}^2 + 2a_y \Delta y$. At the top $v_y = 0$, $v_{y0} = v_0 \sin \theta$ and $a_y = -g$, so $0 = v_0^2 \sin^2 \theta - 2gh$ and $v_0 = \sqrt{2gh} / \sin \theta$.

Common mistake

Saying the speed is zero at the top. Only v_y is zero there. The ball still moves sideways at its launch v_x , so its speed is at its smallest, not zero. In the 37° example, the speed at the top is still 16 m/s.

Unit 2 Force and Translational Dynamics

18–23%
of multiple choice

Forces change motion. You draw each force on a free-body diagram and add them in Newton's second law, for gravity, friction, springs and ropes, and for motion in a circle. Unit 2 shares the largest slice of multiple choice with Unit 3, and many free-response questions start with a free-body diagram.

Not on the exam: calculating tension in a rope with mass or a banked curve that needs friction (explain these only in words), and Kepler's first two laws.

2.1 Systems and center of mass

A system is whatever you draw a boundary around, from one block to a whole train. Forces between objects inside it are internal; forces from outside are external. Only external forces change how the system's center of mass moves, so you can often treat a system as one object at its center of mass.

On the reference sheet

$$x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}$$

Worked example

A 2.0 kg mass sits at $x = 0$ and a 6.0 kg mass at $x = 4.0$ m. Where is the center of mass?

1 $x_{\text{cm}} = (2.0 \cdot 0 + 6.0 \cdot 4.0)/8.0$

3.0 m: 1.0 m from the 6.0 kg mass, 3.0 m from the 2.0 kg.

Common mistake

Calling a force internal or external before choosing the system. Friction between a block and the cart it slides on is internal to the block-and-cart system, but external to the block alone. For the pair, the two friction forces cancel.

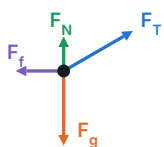
2.2 Forces and free-body diagrams

A force is a push or pull that one object exerts on another, so every force needs an object that exerts it. Gravity is the only one in this course that acts at a distance.

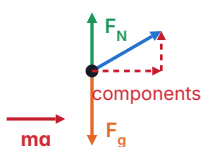
On a free-body diagram the object is a dot. Give each force its own labeled, straight arrow that starts on the dot and points the way the force acts; forces in the same direction go side by side. Leave out components, velocity and any net-force or " ma " arrow.

Force	Exerted by	Direction
F_g , gravity	Earth	down, toward Earth's center; $F_g = mg$
F_N , normal	a surface	perpendicular to the surface, pushing
F_T , tension	a string or rope	along the string, pulling
F_f , friction	a surface	along the surface, against slipping
F_s , spring	a spring	back toward its relaxed length

Right



Wrong



Both show a box pulled by a rope across a rough floor. The wrong one splits the rope's pull into components, adds an " ma " arrow and leaves out friction.

Common mistake

Adding a "force of the throw" to a ball in flight. Once the ball leaves your hand, your hand exerts nothing on it. Only Earth pulls on it, so the one arrow is F_g , down, even on the way up.

2.3 Newton's third law

Forces come in pairs because a force is an interaction. When A pushes or pulls on B, B pushes or pulls on A with a force of the same size and type, in the opposite direction, at the same instant. The two forces act on different objects, so they never cancel on one free-body diagram, and equal forces can still give very different accelerations.

Not on the reference sheet $\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$

Worked example

A 2000 kg truck and a 1000 kg car collide, and at one instant the truck pushes the car with 2.0×10^4 N. Find each one's acceleration.

- The car pushes back on the truck with 2.0×10^4 N.
- Truck: $a = 2.0 \times 10^4 / 2000$. Car: $a = 2.0 \times 10^4 / 1000$.

10 m/s² for the truck and 20 m/s² for the car. The car has half the mass, so the same force gives it twice the acceleration.

Common mistake

Calling a book's weight and the table's normal force on it a third-law pair. Both act on the book, and they come from two different interactions, so they are not partners. In an accelerating elevator they aren't even equal. The partner of Earth's pull on the book is the book's pull on Earth.

2.4 Newton's first law

When the net force on an object is zero, its velocity stays constant: same speed, same direction. Being at rest is just the case $v = 0$. A car cruising at a steady 25 m/s on a straight road has zero net force, because the road's forward push on its tires exactly balances drag. This holds in frames that aren't accelerating, which every exam question uses unless it says otherwise.

Not on the reference sheet $\sum \vec{F} = 0$ exactly when $\vec{v}$ is constant

Worked example

A 2.0 kg sign hangs at rest from two cables, each 37° above the horizontal. Find the tension in each.

- Horizontal: the cables pull equally hard in opposite directions, so the tensions match.
- Vertical: $2F_T \sin 37^\circ = mg$, so $2F_T (0.60) = 20$ N.

$F_T = 17$ N in each cable.

Common mistake

Thinking motion needs a net force. "The elevator moves up, so the floor pushes harder than mg " is wrong when it moves at a constant velocity: the floor, and a scale under you, then push up with exactly mg , going up, going down or standing still. It reads more than mg only while the acceleration points up.

2.5 Newton's second law

The net force sets the acceleration: add up every force on the object or system and divide by its mass. The acceleration points along the net force, which need not be the way the object moves. A cart rolling right while it slows down has a net force pointing left.

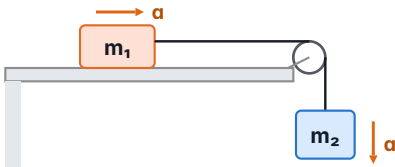
On the reference sheet
$$a_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

Setting up any force problem

- 1 Pick the object or system the question names, and draw its free-body diagram.
- 2 Choose a positive direction along the acceleration. For objects joined by a string, let positive follow the string around the pulley.
- 3 Write $\sum F = ma$ along that axis, one term for each force, with its sign.
- 4 Solve in symbols first, check that the answer behaves sensibly in an extreme case, then put in numbers.

Worked example

A 2.0 kg cart (m_1) on a level, frictionless table is pulled by a string over an ideal pulley to a 1.0 kg hanging block (m_2). Find the acceleration and the tension.



- 1 Both objects as one system, along the string: only the block's weight pulls, so $m_2 g = (m_1 + m_2) a$.
- 2 $a = \frac{(1.0)(10)}{2.0 + 1.0} = 3.3 \text{ m/s}^2$. Cart alone: $F_T = m_1 a = (2.0)(3.33)$.
- 3 Check: if the cart had almost no mass, a would approach g , free fall.

$a = 3.3 \text{ m/s}^2$ and $F_T = 6.7 \text{ N}$, less than the block's 10 N weight.

Common mistake

Setting the string's tension equal to the hanging block's weight. That is true only if the block's velocity is constant. Here the block speeds up on the way down, so the net force on it points down and $F_T < m_2 g$: 6.7 N, not 10 N.

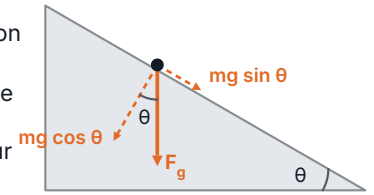
The normal force is not always mg . Find it from the forces perpendicular to the surface each time:

Level floor, nothing else pushing up or down	$F_N = mg$
Pulled by a rope at θ above the horizontal	$F_N = mg - F_T \sin \theta$
On a ramp of angle θ	$F_N = mg \cos \theta$
In an elevator with acceleration a upward	$F_N = m(g + a)$

Worked derivation

Blocks of mass m_1 and m_2 ($m_2 > m_1$) hang on either side of an ideal pulley. Starting with Newton's second law, derive their acceleration. Take both blocks as one system, with positive along the string toward m_2 : $m_2 g - m_1 g = (m_1 + m_2) a$, so $a = (m_2 - m_1)g / (m_1 + m_2)$. The answer may use only m_1 , m_2 and g ; one that still contains F_T is not finished.

On a ramp, tilt the axes: x along the slope, y perpendicular to it. Gravity still points straight down on the diagram; in your equations it splits into $mg \sin \theta$ down the slope and $mg \cos \theta$ into it. The angle between F_g and the perpendicular equals the ramp angle.



Worked example

A 2.0 kg block slides down a frictionless 37° ramp. Find the normal force and the acceleration.

- 1 Perpendicular: $F_N = mg \cos 37^\circ = (20)(0.80) = 16 \text{ N}$.
- 2 Along the slope: $mg \sin 37^\circ = ma$, so $a = g \sin 37^\circ = (10)(0.60)$.

$F_N = 16 \text{ N}$; $a = 6.0 \text{ m/s}^2$ down the slope, for any mass.

2.6 Gravitational force

Any two masses attract each other with forces along the line between their centers, a third-law pair. The distance r runs center to center, so a satellite a height h above a planet of radius R has $r = R + h$. The gravitational field $g = GM/r^2$ is the force on each kilogram: 9.8 N/kg at Earth's surface, which exam questions round to 10 N/kg. Your mass is the same everywhere; your weight, mg , depends on where you are.

On the reference sheet
$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2} \quad G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

Not on the reference sheet
$$g = \frac{GM}{r^2} \quad F_g = mg$$

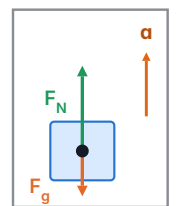
Worked example

Planet X has 3 times Earth's mass and 3 times its radius. What does a 10 kg rock weigh there?

- 1 Using $g = GM/R^2$, $g_X = (10 \text{ N/kg})(3/3^2) = 3.3 \text{ N/kg}$.

33 N, a third of its weight on Earth.

A scale reads the normal force, not mg . In an elevator, with up as positive, $F_N - mg = ma$, so $F_N = m(g + a)$. The reading depends on which way the acceleration points, not which way the elevator moves. Feeling weightless means gravity is the only force on you, as in free fall or orbit.



Worked example

A 60 kg student stands on a scale in an elevator that is moving up but slowing at 2.0 m/s^2 . What does the scale read?

- 1 Up is positive. Moving up while slowing means the acceleration points down: $a = -2.0 \text{ m/s}^2$.
- 2 $F_N = m(g + a) = 60(8.0)$

480 N, less than the student's 600 N weight.

Common mistake

Using the height above the surface as r . A satellite 400 km above Earth (radius $6.37 \times 10^6 \text{ m}$) is $6.77 \times 10^6 \text{ m}$ from the center, and that is the r that goes in the formula. Using $4.0 \times 10^5 \text{ m}$ makes the force about 290 times too big.

2.7 Kinetic and static friction

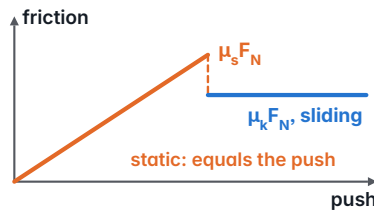
When two surfaces slide past each other, kinetic friction acts against the sliding with a size of $\mu_k F_N$. When they don't slide, static friction is only as large as it needs to be to stop them slipping, up to a limit of $\mu_s F_N$. For the same pair of surfaces μ_s is usually larger than μ_k . Friction opposes slipping, not motion: when you walk, static friction on your shoe points forward.

On the reference sheet $|\vec{F}_f| \leq \mu \vec{F}_N$

Not on the reference sheet sliding: $F_f = \mu_k F_N$

not sliding: $F_f \leq \mu_s F_N$

The friction on a block as you push it harder and harder: it slides once the push passes $\mu_s F_N$.



Worked example

A 2.0 kg block rests on a floor with $\mu_s = 0.40$ and $\mu_k = 0.30$. Find the friction and the acceleration when you push sideways with 5.0 N, and then with 10 N.

- 1 $F_N = mg = 20$ N, so static friction can give at most $(0.40)(20) = 8.0$ N.
- 2 5.0 N is less than 8.0 N: the block stays put, and friction is 5.0 N.
- 3 10 N is more: it slides, friction is $(0.30)(20) = 6.0$ N, and $a = (10 - 6.0)/2.0$.

5.0 N with $a = 0$; then 6.0 N with $a = 2.0$ m/s².

Common mistake

Writing $F_f = \mu_s F_N$ for an object at rest. That is the most static friction can give, not what it actually gives. Under the 5.0 N push above, friction is 5.0 N, not 8.0 N. Find static friction from the force balance.

A block that is just about to slip on a ramp tilted at θ has $mg \sin \theta = \mu_s mg \cos \theta$, so $\mu_s = \tan \theta$, whatever its mass.

2.8 Spring forces

An ideal spring pulls or pushes toward its relaxed length with a force proportional to how far it is stretched or squeezed. Δx is measured from the relaxed length, and the minus sign in $\vec{F}_s = -k\Delta\vec{x}$ means the force points back toward it. A larger spring constant k means a stiffer spring.

On the reference sheet $\vec{F}_s = -k\Delta\vec{x}$

In the lab, hang several masses, measure each stretch, and graph the force mg against the stretch Δx . The slope of the best-fit line is k . A 0.50 kg mass hanging at rest on a 25 N/m spring stretches it $\Delta x = mg/k = 5.0/25 = 0.20$ m.

Common mistake

Using the spring's length instead of its stretch. A 0.30 m spring pulled out to 0.36 m has $\Delta x = 0.06$ m, so with $k = 500$ N/m it pulls with 30 N, not 180 N.

2.9 Circular motion

An object moving in a circle at constant speed is still accelerating, because its direction keeps changing. The acceleration points to the center of the circle. Real forces have to supply the inward net force, mv^2/r : tension, gravity, the normal force or friction. "Centripetal force" is a name for that inward total, never an extra arrow on the diagram.

On the reference sheet $a_c = \frac{v^2}{r}$ $T = \frac{1}{f}$

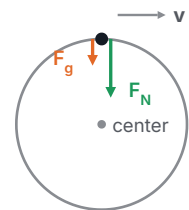
Not on the reference sheet $\sum F_{\text{toward center}} = \frac{mv^2}{r}$ $v = \frac{2\pi r}{T}$

Situation	What supplies the inward force
Car on a flat curve	static friction from the road
Ball on a string, level circle	the horizontal part of the tension
Car on a frictionless banked curve	the horizontal part of the normal force
Satellite or moon in orbit	gravity alone
Top of a loop	gravity and the normal force, both down

Worked example

A 0.50 kg cart moves at 8.0 m/s through the top of a loop of radius 2.0 m. Find the normal force, and the slowest speed at which it stays on the track.

- 1 The inward net force needed is $mv^2/r = (0.50)(64)/2.0 = 16$ N.
- 2 Gravity gives 5.0 N of it, so the track pushes down with $16 - 5.0$.
- 3 At the slowest speed the track pushes with zero force, so $mg = mv^2/r$ and $v = \sqrt{gr} = \sqrt{20}$.



$F_N = 11$ N, down; slowest speed 4.5 m/s.

Worked example

A 1200 kg car takes a flat curve of radius 50 m at 15 m/s. How much friction does it need, and what is the smallest μ_s that works?

- 1 Inward force needed: $mv^2/r = (1200)(15^2)/50 = 5400$ N, all from static friction.
- 2 $\mu_s mg \geq mv^2/r$, so $\mu_s \geq v^2/(gr) = 225/500$.

5400 N of friction; μ_s at least 0.45, for any mass of car.

Common mistake

Drawing the normal force up at the top of a loop, or adding a "centripetal force" arrow. At the top the track is above the cart, so it pushes down, toward the center. Draw F_g and F_N both pointing down, side by side, and nothing else.

Worked derivation

For a circular orbit of radius r around a planet of mass M , gravity is the only inward force: $\frac{GMm}{r^2} = \frac{m}{r} \left(\frac{2\pi r}{T} \right)^2$, so

$T^2 = \frac{4\pi^2}{GM} r^3$. A graph of T^2 against r^3 for several moons is a straight line with slope $4\pi^2/GM$, so the planet's mass is $M = 4\pi^2/(G \times \text{slope})$.

Unit 3 Work, Energy, and Power

18–23%
of multiple choice

Energy lets you connect two moments in a motion without knowing what happened in between. This unit covers kinetic and potential energy, the work forces do, conservation of energy and power. In the free-response section, expect energy bar charts, graphs of energy against position and derivations that start from conservation of energy.

Not on the exam: heat flowing between systems, which is AP Physics 2.

3.1 Translational kinetic energy

Kinetic energy is the energy of motion. It depends on speed but not on direction, so it is a scalar and never negative. It grows with the square of the speed: twice the speed means four times the kinetic energy.

On the reference sheet $K = \frac{1}{2}mv^2$

Worked example

A 1500 kg car speeds up from 10 m/s to 20 m/s. How much kinetic energy does it gain?

$$1 \quad K_f = \frac{1}{2}(1500)(10)^2 = 75 \text{ kJ}; K_f = \frac{1}{2}(1500)(20)^2 = 300 \text{ kJ}.$$

225 kJ: three times the 75 kJ that got it to 10 m/s.

Common mistake

Thinking twice the speed means twice the energy. A ball at 8 m/s has four times the kinetic energy it has at 4 m/s, so the same braking force needs four times the distance to stop it.

3.2 Work

Work is the energy a force adds to or removes from a system when it pushes or pulls something through a distance. Only the part of the force along the displacement counts, so θ is the angle between the force and the displacement. Work is positive when the force has a part along the motion, negative when it points against it, and zero when it is perpendicular or its point of contact doesn't move. Add the work of every force, with signs, and you get the change in kinetic energy. For a force that changes, the work is the area under the force-position graph.

On the reference sheet $W = F_{\parallel}d = Fd \cos \theta$

$$\Delta K = \sum W_i = \sum F_{\parallel,i} d_i$$

Worked example

A rope pulls a 20 kg sled from rest 15 m across level snow with 100 N at 37° above the horizontal; $\mu_k = 0.20$. Find the work done by each force and the final speed.

- Rope: $W = (100)(15) \cos 37^\circ = (100)(15)(0.80) = +1200 \text{ J}$.
- The rope's upward part, $100 \sin 37^\circ = 60 \text{ N}$, takes weight off the snow: $F_N = 200 - 60 = 140 \text{ N}$, friction is $(0.20)(140) = 28 \text{ N}$ and $W = -28(15) = -420 \text{ J}$.
- Gravity and the normal force do no work. $\Delta K = 1200 - 420 = 780 \text{ J} = \frac{1}{2}(20)v^2$.

$$v = \sqrt{78} = 8.8 \text{ m/s}.$$

Common mistake

Using the ramp's length to find gravity's work. Only the vertical drop counts. A 1.5 kg block that slides 4.0 m down a 30° ramp drops $4.0 \sin 30^\circ = 2.0 \text{ m}$, so gravity does $(1.5)(10)(2.0) = 30 \text{ J}$ of work, not 60 J.

3.3 Potential energy

Potential energy is stored in the arrangement of a system whose parts pull or push on each other: an object and Earth, or an object and a spring. A single object has none on its own, so say "the ball-Earth system," not "the ball." Near Earth's surface use $\Delta U_g = mg\Delta y$; only changes matter, so put the zero wherever is convenient. A spring's potential energy is zero at its relaxed length and never negative. When the distance between two objects changes a lot, as for a satellite, use $U_G = -Gm_1m_2/r$, which is zero when they are infinitely far apart and negative everywhere else.

On the reference sheet $\Delta U_g = mg\Delta y$ $U_s = \frac{1}{2}k(\Delta x)^2$

$$U_G = -\frac{Gm_1m_2}{r}$$

Not on the reference sheet $W_{\text{gravity or spring}} = -\Delta U$

When gravity or a spring does positive work, the system's potential energy drops by the same amount. That is why you can count gravity either as work done on an object or as a change in U_g , but never as both.

Worked example

A spring with $k = 400 \text{ N/m}$ is squeezed 0.10 m, then 0.20 m. How much energy does it store each time?

$$1 \quad \frac{1}{2}(400)(0.10)^2 = 2.0 \text{ J and } \frac{1}{2}(400)(0.20)^2 = 8.0 \text{ J}.$$

Twice the squeeze stores four times the energy.

Common mistake

Squaring only part of the squeeze. A spring compressed by $4D$ stores $\frac{1}{2}k(4D)^2 = 8kD^2$. Writing $\frac{1}{2}k \cdot 4D^2 = 2kD^2$ loses a factor of four, and a derivation built on it can't recover.

Worked example

A 1000 kg satellite moves from Earth's surface to a distance $2R$ from Earth's center. Earth has $R = 6.37 \times 10^6 \text{ m}$ and $M = 5.97 \times 10^{24} \text{ kg}$. How much does U_G change?

$$1 \quad \Delta U_G = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R}$$

$$2 \quad = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})(1000)}{2(6.37 \times 10^6)}$$

It rises by $3.1 \times 10^{10} \text{ J}$. Using mgR would double that, because g weakens with height.

Common mistake

Thinking a change from $-5 \times 10^9 \text{ J}$ to $-2 \times 10^9 \text{ J}$ is a decrease. A negative U_G only means the zero was put at infinite separation. $-2 \times 10^9 \text{ J}$ is the larger number, so U_G rose by $3 \times 10^9 \text{ J}$, energy that something had to supply.

3.4 Conservation of energy

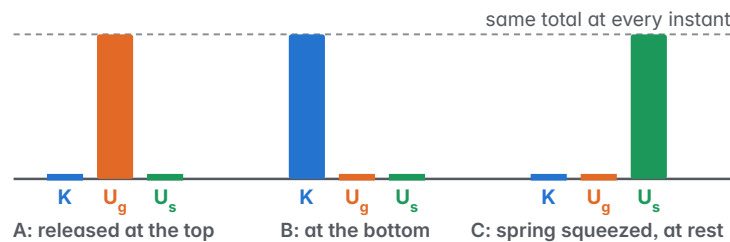
Energy is never created or destroyed. A system's total energy stays constant when no energy crosses its boundary. If there is also no conversion between mechanical and internal energy, its mechanical energy, $K + U$, stays constant too. Friction turns mechanical energy into thermal energy: sliding a distance d against kinetic friction F_f on a fixed surface converts $F_f d$, where d is the length of the path, not the displacement.

Energy links two positions without needing the time between them. When a question asks for a time, an acceleration or a force at one instant, use Newton's laws and kinematics instead.

Not on the reference sheet $K_i + U_i = K_f + U_f$

with friction: $K_i + U_i = K_f + U_f + F_f d$

Energy bar charts



A block slides from rest down a smooth ramp, then squeezes a spring on a smooth floor until it stops. The system is the block, Earth and the spring.

To earn the points, start every bar at the zero line. If an energy is zero, draw a short line on the zero line in its place, not a gap. Follow the scale, and keep the total the same at every instant when nothing outside does work. Include only the energies your named system has: if Earth is not in the system, there is no U_g bar.

Writing the justification

To justify constant mechanical energy, name the system and say why: "No outside force does work on the block–Earth system and the track is frictionless, so its $K + U_g$ stays constant: the U_g it loses equals the K it gains."

Worked example

A 2.0 kg block slides from rest down a frictionless track that drops 1.8 m, then 3.0 m across a floor with $\mu_k = 0.20$. Find its final speed and how far it would slide on that floor before stopping.

- Energy from the drop: $mgh = (2.0)(10)(1.8) = 36$ J.
- Turned to thermal energy on the floor: $\mu_k mgd = (0.20)(2.0)(10)(3.0) = 12$ J.
- $K_f = 36 - 12 = 24$ J = $\frac{1}{2}(2.0)v^2$. To stop, $mgh = \mu_k mgd$, so $d = h/\mu_k$.

$v = 4.9$ m/s; it would stop after $1.8/0.20 = 9.0$ m, whatever its mass.

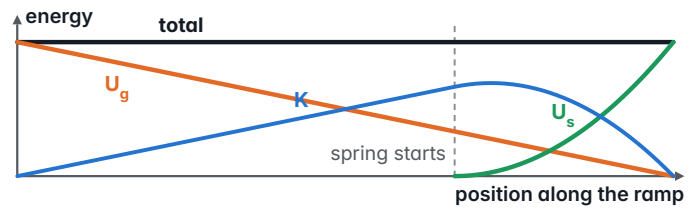
Worked example

A 0.50 kg ball on a 0.80 m string is released from rest with the string horizontal. Find the tension at the bottom of the swing.

- Energy gives the speed (the tension does no work): $mgL = \frac{1}{2}mv^2$, so $v^2 = 2gL = 16$ m²/s².
- Forces at the bottom, with up (toward the center) positive: $F_T - mg = mv^2/L$, so $F_T = 5.0 + (0.50)(16)/0.80$.

15 N, three times the ball's weight, for any string length.

Energy against position



Here the spring sits partway down the ramp itself. K is whatever is left of the constant total.

Common mistake

Assuming the block slows down the moment it touches the spring. It is still dropping, so it keeps losing U_g , while the barely squeezed spring stores only a little U_s . Its K keeps growing for a while after contact, until the spring pushes back harder than gravity pulls it down the slope.

Worked derivation

A block slides from rest at height h down a frictionless track into a loop of radius R . Find the smallest h that keeps it on the track at the top. At the top, gravity and the normal force both point toward the center; just staying on means $F_N = 0$, so $mg = mv^2/R$ and $v^2 = gR$. Energy from release to the top: $mgh = \frac{1}{2}mv^2 + mg(2R)$, so $mgh = \frac{1}{2}mgR + 2mgR$ and $h = 2.5R$.

In the lab, to find μ_k with only a meterstick, release a block from several heights h on a smooth ramp and measure how far d it slides on the rough floor. Since $mgh = \mu_k mgd$, h against d is a straight line with slope μ_k .

3.5 Power

Power is how fast energy is transferred or changed from one form to another, in joules per second, or watts. The same work done in half the time takes twice the power. At any instant a force delivers power $F_{\parallel} v$, so a steady force delivers more power as the object speeds up.

On the reference sheet

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t} \quad P_{\text{inst}} = F_{\parallel} v = F v \cos \theta$$

Worked example

A motor lifts a 40 kg load 3.0 m at a steady speed in 6.0 s. What is its power output?

- $W = mg\Delta y = (40)(10)(3.0) = 1200$ J, so $P = 1200/6.0$.
- Check with $P = Fv$: $F = mg = 400$ N and $v = 3.0/6.0 = 0.50$ m/s.

200 W either way.

Common mistake

Counting gravity twice. With Earth inside your system, gravity appears as U_g ; with Earth outside, it appears as the work gravity does on the object, $-mg\Delta y$. Use one or the other. A 0.50 kg ball that falls 4.0 m gains 20 J of kinetic energy, not 40 J.

Common mistake

Assuming the higher-power device always uses more energy. Power is a rate, so energy = power $\times$ time. A 10 W charger running for 2.0 hours (7200 s) and a 60 W bulb lit for 20 minutes (1200 s) both use 72,000 J.

Unit 4 Linear Momentum

10–15%
of multiple choice

Momentum stays the same when objects collide or push apart, as long as nothing outside pushes on them. Free-response questions ask for momentum arrows, momentum–time graphs and the speeds after a collision, and then often ask what happened to the kinetic energy.

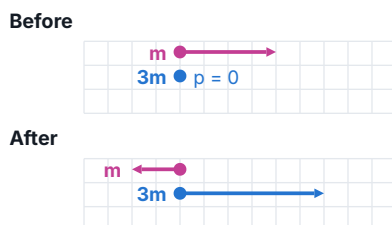
Not on the exam: calculations for objects whose mass changes, like rockets, and solving simultaneous equations for two-dimensional collisions (setting them up and predicting changes are tested).

4.1 Linear momentum

Momentum is mass times velocity. It is a vector pointing the way the object moves, so momenta add with signs. A system's momentum is the sum of its parts' momenta, which also equals its total mass times the velocity of its center of mass.

On the reference sheet $\vec{p} = m\vec{v}$ $\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i}$

A light puck (m) hits a heavy one ($3m$) at rest and bounces back. Arrow length is mv : after the hit both pucks move at the same speed, so the heavy puck's arrow is three times as long. The total is 4 units before and $-2 + 6 = 4$ units after.



Worked example

A 2.0 kg cart moves at +3.0 m/s toward a 3.0 kg cart moving at -2.0 m/s. Find the system's momentum and kinetic energy.

$$1 \quad p = (2.0)(+3.0) + (3.0)(-2.0) = 0$$

$$2 \quad K = \frac{1}{2}(2.0)(3.0)^2 + \frac{1}{2}(3.0)(2.0)^2 = 9.0 + 6.0$$

The momentum is zero but $K = 15$ J. Opposite momenta cancel; kinetic energy is never negative, so it can't.

4.2 Change in momentum and impulse

A net force acting over a time changes momentum. The impulse, average force times time, equals the change in momentum. On a force–time graph the impulse is the area under the curve; on a momentum–time graph the slope is the net force. Making a stop last longer, as an airbag or a soft catch does, leaves Δp the same but lowers the average force.

On the reference sheet $\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} = m\vec{a}$ $\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta \vec{p}$

Worked example

A 0.20 kg ball hits a wall at +5.0 m/s and bounces straight back at 4.0 m/s. Contact lasts 0.030 s. Find the impulse and the wall's average force on the ball.

$$1 \quad \Delta p = m(v - v_0) = 0.20(-4.0 - 5.0) = -1.8 \text{ N}\cdot\text{s}$$

$$2 \quad F_{\text{avg}} = \Delta p / \Delta t = -1.8 / 0.030$$

$J = -1.8 \text{ N}\cdot\text{s}$ and $F_{\text{avg}} = -60 \text{ N}$, pointing away from the wall.

Common mistake

Thinking impulse points the way the object moves. A safety net catching a falling stunt performer pushes up the whole time the performer moves down, so its impulse points up. Impulse points along the average force, the direction of the change in momentum, not of the momentum itself.

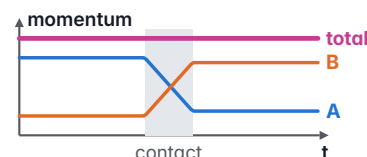
4.3 Conservation of linear momentum

Forces between parts of a system are third-law pairs acting for the same time, so their impulses cancel. Only a net outside force can change the system's total momentum. If it is zero, or too small to matter during a brief collision, the total before equals the total after. A falling ball gains momentum; the ball–Earth system doesn't.

Not on the reference sheet $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

i means before the collision, f after

Two carts collide on a frictionless track. Each one's momentum changes by an equal and opposite amount, so the total stays flat.



Worked example

Two skaters, 50 kg and 75 kg, push apart from rest. The 75 kg skater moves off at +2.0 m/s. Find the other skater's velocity.

$$1 \quad 0 = (50)v + (75)(+2.0), \text{ so } v = -150/50.$$

-3.0 m/s; their kinetic energy rose from 0 to 375 J.

Writing the justification

"Momentum is conserved" alone earns nothing. Name the system and the forces: "The system is the two skaters. Their pushes on each other are internal, equal and opposite for the same time, so their impulses cancel. Friction from the ice is negligible, so no net outside force acts horizontally, and the total momentum stays zero."

4.4 Elastic and inelastic collisions

Momentum is conserved in every collision with negligible outside force. Whether kinetic energy is conserved tells you which kind of collision it is. In an elastic collision the total kinetic energy is unchanged. In an inelastic one some becomes internal energy and sound; objects that stick lose the most. An explosion gains kinetic energy from stored energy.

Worked example

A 2.0 kg cart at 3.0 m/s hits a 1.0 kg cart at rest, and they stick. Find their speed and the kinetic energy lost.

$$1 \quad \text{Momentum first: } (2.0)(3.0) = (3.0)v, \text{ so } v = 2.0 \text{ m/s.}$$

$$2 \quad K \text{ before: } \frac{1}{2}(2.0)(3.0)^2 = 9.0 \text{ J. After: } \frac{1}{2}(3.0)(2.0)^2 = 6.0 \text{ J.}$$

2.0 m/s; 3.0 J of kinetic energy lost.

Common mistake

Using conservation of kinetic energy for objects that stick. Setting $\frac{1}{2}(2.0)(3.0)^2 = \frac{1}{2}(3.0)v^2$ gives 2.45 m/s, which is wrong because kinetic energy is lost. Use momentum to find the shared speed, then compare the kinetic energies to find the loss.

Unit 5 Torque and Rotational Dynamics

10–15%
of multiple choice

This unit is Units 1 and 2 again for objects that turn. Torque plays the part of force and rotational inertia the part of mass. Free-response questions have used pulleys that have mass and beams held in balance.

Not on the exam: the direction of torque as a vector (you only say clockwise or counterclockwise), rotation about two axes at once, and memorizing the rotational inertia of rods, disks or spheres, which the question gives you.

5.1 Rotational kinematics

Turning has its own versions of position, velocity and acceleration: the angle θ in radians, the angular velocity ω and the angular acceleration α . Call one direction, usually counterclockwise, positive. When α is constant, three equations from the reference sheet work just like the Unit 1 equations, and when ω and α have the same sign, the object spins faster.

On the reference sheet $\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

Not on the reference sheet $\omega_{\text{avg}} = \Delta\theta/\Delta t$ $\alpha_{\text{avg}} = \Delta\omega/\Delta t$

$$1 \text{ rev} = 2\pi \text{ rad} = 360^\circ$$

Worked example

A fan turning at 30 rad/s slows steadily and stops after turning through 50 rad. Find α and the stopping time.

- No time given: $0 = 30^2 + 2\alpha(50)$, so $\alpha = -900/100 = -9.0 \text{ rad/s}^2$.
- $t = \Delta\omega/\alpha = (0 - 30)/(-9.0)$.

$\alpha = -9.0 \text{ rad/s}^2$, opposite to the spin; it stops after 3.3 s.

Common mistake

Putting revolutions or degrees into the equations. Convert to radians first: a wheel turning at 300 revolutions per minute has $\omega = 300 \times 2\pi/60 = 31.4 \text{ rad/s}$, not 300.

5.2 Connecting linear and rotational motion

Every point on a rigid object turns through the same angle with the same ω and α . Points farther from the axis travel farther and faster, in proportion to r , and their tangential acceleration a_T , along the path, is $r\alpha$. Each point also has a centripetal acceleration toward the axis, $v^2/r = r\omega^2$, even when ω is constant.

On the reference sheet $v = r\omega$ $a_T = r\alpha$ $s = r\theta$

Not on the reference sheet $a_c = r\omega^2$

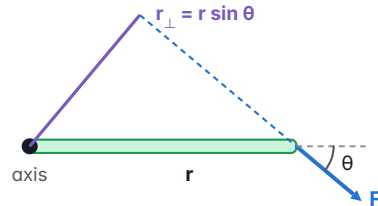
Common mistake

Saying the outer point "spins faster." Two children 1.0 m and 2.0 m from the center of a merry-go-round share the same ω . The outer one has twice the speed and twice the centripetal acceleration, because both grow with r .

5.3 Torque

Torque measures how strongly a force turns an object about an axis. It depends on the size of the force, how far from the axis it acts, and its angle. In $\tau = rF \sin \theta$, r runs from the axis to the point where the force acts, and θ is the angle between r and the force. Equivalently, multiply the force by its lever arm $r_\perp$, the perpendicular distance from the axis to the force's line. A force whose line passes through the axis makes no torque.

On the reference sheet $\tau = r_\perp F = rF \sin \theta$



Worked example

A uniform 2.0 kg rod, 1.2 m long, is pivoted at one end and held 30° below the horizontal. Find the torque of its weight about the pivot.

- The weight, 20 N, acts at the center: $r = 0.60 \text{ m}$ from the pivot.
- The rod points 30° below horizontal and the weight points straight down, so the angle between them is 60° .
- $\tau = (0.60)(20) \sin 60^\circ$.

10 N·m. Using $\sin 30^\circ$ gives 6.0 N·m, which is wrong.

Common mistake

Using the angle the object makes with the horizontal as θ . θ is the angle between r and the force, so draw both from the same point, tail to tail. The rod above is 30° below the horizontal, but θ is 60° .

5.4 Rotational inertia

Rotational inertia is to turning what mass is to moving in a line: the more of it, the harder an object is to spin up or slow down. It depends on the mass and on how far that mass sits from the axis, with the distance squared. Mass on the axis adds nothing.

On the reference sheet $I = \sum m_i r_i^2$ $I' = I_{\text{cm}} + Md^2$

The two axes must be parallel and a distance d apart.

Same mass and radius: the hoop has all its mass at the rim, so it has the most rotational inertia. The question gives you these formulas when you need them.



hoop
 MR^2



disk
 $\frac{1}{2}MR^2$



solid sphere
 $\frac{2}{5}MR^2$

Worked example

A uniform 3.0 kg rod is 2.0 m long, with $I_{\text{cm}} = \frac{1}{12}ML^2$ about its center. Find I about one end.

- $I_{\text{cm}} = \frac{1}{12}(3.0)(2.0)^2 = 1.0 \text{ kg}\cdot\text{m}^2$, and the end is $d = 1.0 \text{ m}$ away: $I' = 1.0 + (3.0)(1.0)^2$.

4.0 $\text{kg}\cdot\text{m}^2$, four times I_{cm} .

Common mistake

Forgetting the square. A small mass moved from 0.20 m to 0.60 m from the axis is three times as far out, so it adds nine times as much rotational inertia, not three times.

5.5 Rotational equilibrium

An object stays at rest only if the net force and the net torque on it are both zero. When an object is in equilibrium, the torques cancel about every point, so take torques about the point where an unknown force acts: that force has no lever arm there and drops out. Then use $\sum F = 0$ for the rest. A wheel spinning at a steady rate is also in rotational equilibrium.

Not on the reference sheet $\sum F = 0$ and $\sum \tau = 0$

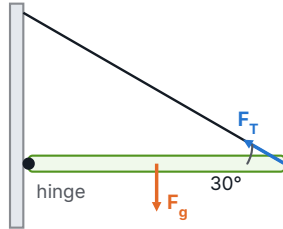
Worked example

A uniform 2.0 kg beam is hinged to a wall and held level by a string tied to its far end at 30° to the beam. Find the tension.

- 1 Torques about the hinge: the weight $F_g = Mg$ acts at the middle, so $F_T L \sin 30^\circ = Mg \frac{L}{2}$. The length cancels.

- 2 $F_T = \frac{Mg}{2 \sin 30^\circ} = \frac{20}{2(0.50)}$

20 N. A shallower string needs more tension.



5.6 Newton's second law in rotational form

A net torque gives an angular acceleration. Twice the net torque gives twice the angular acceleration; twice the rotational inertia gives half. For a pulley that has mass, write Newton's second law for the hanging block and the rotational version for the pulley, then join them with $a = R\alpha$.

On the reference sheet

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}} = \frac{\tau_{\text{net}}}{I_{\text{sys}}}$$

Worked example

A 1.0 kg block hangs from a string wrapped around a pulley ($R = 0.20$ m, $I = 0.040$ kg·m²) that turns freely. Find the block's acceleration and the tension.

- 1 Block: $mg - F_T = ma$. Pulley: $F_T R = I\alpha$. No slipping: $a = R\alpha$.
- 2 Replace the angular acceleration with a/R : $F_T = Ia/R^2$, so $mg = (m + I/R^2)a$ and $a = 10/(1.0 + 1.0)$.
- 3 $F_T = m(g - a) = (1.0)(10 - 5.0)$.

$a = 5.0$ m/s² and $F_T = 5.0$ N, half the block's weight.

Common mistake

Using the block's weight for the torque on the pulley. The string's tension turns the pulley, and it is less than mg whenever the block speeds up. The torque is $F_T R = 1.0$ N·m, not $mgR = 2.0$ N·m.

Common mistake

Expecting the largest angular acceleration where the angular speed is largest. A rod released from horizontal on a hinge has its largest α at release, where gravity's lever arm is longest, and its largest ω at the bottom, where the torque is zero.

Unit 6 Energy and Momentum of Rotating Systems

5–8%
of multiple choice

This unit applies the energy and momentum ideas of Units 3 and 4 to spinning objects, rolling objects and satellites in orbit. Free-response questions have used a disk rolling down a ramp and a string pulled off a spinning toy.

Not on the exam: the direction of angular momentum in three dimensions, rolling friction, Kepler's first and second laws by name, and equations for rolling while slipping (you explain that in words).

6.1 Rotational kinetic energy

A spinning object has kinetic energy even if its center never moves, because every piece of it is moving. Something that rolls has both kinds: $\frac{1}{2}Mv_{\text{cm}}^2$ for its center moving along, plus $\frac{1}{2}I_{\text{cm}}\omega^2$ for its spin.

On the reference sheet $K = \frac{1}{2}I\omega^2$

Not on the reference sheet $K_{\text{total}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2$

Worked example

A flywheel with $I = 2.0$ kg·m² turns at 300 revolutions per minute. Find its kinetic energy.

- 1 $\omega = 300 \times 2\pi/60 = 31.4$ rad/s, so $K = \frac{1}{2}(2.0)(31.4)^2$.

About 990 J. Putting in 300 instead gives 90,000 J.

Common mistake

Counting energy twice for something that turns about a fixed pivot, like a door or a swinging rod. There, $\frac{1}{2}I_{\text{pivot}}\omega^2$ is already all of its kinetic energy. Add $\frac{1}{2}Mv_{\text{cm}}^2$ only when you use I_{cm} , as for rolling.

6.2 Torque and work

A torque does work only while the object turns through an angle, in radians. Lean on a stuck bolt and you exert a torque but do no work. The work done by all the torques equals the change in rotational kinetic energy. For a torque that changes, the work is the area under the torque–angle graph.

On the reference sheet $W = \tau\Delta\theta$

Worked derivation

A toy turns freely about a fixed axis. A string wrapped around its axle is pulled with a steady force F through a length ℓ , starting from rest. Find the final ω . The work is $F\ell$, whatever the axle's radius, so $F\ell = \frac{1}{2}I\omega^2$ and $\omega = \sqrt{2F\ell/I}$. Half the rotational inertia gives $\sqrt{2}$ times the angular speed, not twice: with $F = 8.0$ N and $\ell = 0.50$ m, $I = 0.020$ kg·m² gives 20 rad/s and $I = 0.010$ gives 28 rad/s.

Common mistake

Using the rod's whole length as the drop. A uniform rod of length L swinging down from horizontal about one end lowers its center of mass by only $L/2$, so gravity does $MgL/2$ of work.

6.3 Angular momentum and angular impulse

Angular momentum is the turning version of momentum. A rigid object spinning about an axis has $L = I\omega$. An object moving in a straight line also has angular momentum about a point, $rmv \sin \theta$, where θ is the angle between r and v . That equals mv times the perpendicular distance from the point to its path, so it is zero only for points on the path. A net torque acting for a time changes L : the angular impulse, the area under the net torque–time graph, equals ΔL . For constant or average net torque it is $\tau \Delta t$.

On the reference sheet $L = I\omega$ $L = rmv \sin \theta$ $\Delta L = \tau \Delta t$

Worked example

A disk ($I = 0.010 \text{ kg}\cdot\text{m}^2$) and a hoop ($I = 0.020 \text{ kg}\cdot\text{m}^2$), both of radius 0.10 m , start at rest. A string around each is pulled with 5.0 N for 2.0 s . Compare their final angular speeds.

- Same torque, $(0.10)(5.0) = 0.50 \text{ N}\cdot\text{m}$, for the same time: $\Delta L = (0.50)(2.0) = 1.0 \text{ kg}\cdot\text{m}^2/\text{s}$ for each.
- $\omega = L/I$: $1.0/0.010$ for the disk, $1.0/0.020$ for the hoop.

100 rad/s and 50 rad/s: same angular momentum, but the disk spins twice as fast.

Common mistake

Assuming the disk always spins twice as fast as the hoop. That is true for the same torque over the same time, which gives both the same ΔL . Over the same angle, the torque does the same work on each, so they get the same K , and the disk spins only $\sqrt{2} \approx 1.4$ times as fast.

6.4 Conservation of angular momentum

If no net outside torque acts on a system, its total angular momentum stays constant, whatever happens inside it. Pulling mass in toward the axis lowers I , so ω rises. Kinetic energy usually changes even though L doesn't: a skater who pulls in her arms does work and gains kinetic energy, and clay that sticks to a spinning rod loses some.

Not on the reference sheet $I_i\omega_i = I_f\omega_f$ when no net outside torque acts

Worked example

A student on a spinning stool has $I = 4.0 \text{ kg}\cdot\text{m}^2$ and turns at 2.0 rad/s . She pulls in her arms to $I = 1.0 \text{ kg}\cdot\text{m}^2$. Find her new ω and her kinetic energy before and after.

- $(4.0)(2.0) = (1.0)\omega_f$, so $\omega_f = 8.0 \text{ rad/s}$.
- K : $\frac{1}{2}(4.0)(2.0)^2 = 8.0 \text{ J}$ before, $\frac{1}{2}(1.0)(8.0)^2 = 32 \text{ J}$ after.

8.0 rad/s; K quadruples, paid for by the work her arms do.

Common mistake

Using the old rotational inertia after something sticks. If 0.10 kg of clay moving at 6.0 m/s perpendicular to a 1.0 m rod hits its free end and sticks, while the rod is at rest and pivoted at its other end ($I = 0.30 \text{ kg}\cdot\text{m}^2$), its angular momentum is $(1.0)(0.10)(6.0) = 0.60 \text{ kg}\cdot\text{m}^2/\text{s}$. The new inertia is $0.30 + (0.10)(1.0)^2 = 0.40 \text{ kg}\cdot\text{m}^2$, so $\omega = 1.5 \text{ rad/s}$, not 2.0 .

6.5 Rolling

A wheel of radius R rolling without slipping has $v_{\text{cm}} = R\omega$, and its kinetic energy is shared between moving and spinning. On a ramp, static friction supplies the torque that spins it up but does no work, because the contact point doesn't slide. Down the same ramp, an object with more of its mass far from its center rolls more slowly: a frictionless sliding block beats a solid sphere, which beats a disk, which beats a hoop.

On the reference sheet $\Delta x_{\text{cm}} = r\Delta\theta$ $v = r\omega$

Not on the reference sheet $v = \sqrt{\frac{2gh}{1 + I_{\text{cm}}/(MR^2)}}$

(rolling from rest down a drop h)

Each bar shows how much of the lost U_g goes into moving forward rather than spinning.

Block sliding, no friction

100%

Solid sphere, $I_{\text{cm}} = \frac{2}{5}MR^2$

71%

Disk, $I_{\text{cm}} = \frac{1}{2}MR^2$

67%

Hoop, $I_{\text{cm}} = MR^2$

50%

Worked example

A solid disk ($I_{\text{cm}} = \frac{1}{2}MR^2$, given) rolls from rest down a ramp that drops 1.5 m . How fast is it moving at the bottom?

- $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(I_{\text{cm}})(\frac{v}{R})^2 = \frac{3}{4}Mv^2$
- $v^2 = \frac{4}{3}gh = \frac{4}{3}(10)(1.5) = 20 \text{ m}^2/\text{s}^2$.

4.5 m/s, for a disk of any mass or size; a third of its K is spin.

Common mistake

Saying the heavier or bigger ball rolls down faster. Mass and radius cancel: a 1 kg and a 5 kg solid ball rolling from rest down a 1.5 m drop both reach 4.6 m/s . The ratio $I_{\text{cm}}/(MR^2)$ sets the speed, so mass distribution matters.

6.6 Motion of orbiting satellites

In a circular orbit, gravity is the only force and supplies the centripetal force: $GMm/r^2 = mv^2/r$, so $v = \sqrt{GM/r}$. On an elliptical orbit, total energy and angular momentum stay constant while K and U_g trade, so the satellite is fastest nearest the planet. At the nearest and farthest points $L = rmv$, so $r_{\text{near}}v_{\text{near}} = r_{\text{far}}v_{\text{far}}$. To escape, the total energy must reach zero.

Not on the reference sheet circular orbit: $v = \sqrt{GM/r}$

circular orbit: $E = -GMm/2r$ $v_{\text{esc}} = \sqrt{2GM/r}$

Worked example

A planet has $GM = 4.0 \times 10^{14} \text{ m}^3/\text{s}^2$. Find the speed and period of a circular orbit of radius $1.0 \times 10^7 \text{ m}$, and the escape speed from there.

- $v = \sqrt{4.0 \times 10^{14}/1.0 \times 10^7} = 6.3 \times 10^3 \text{ m/s}$; $T = 2\pi r/v$.
- The escape speed is $\sqrt{2}$ times as large.

6.3 km/s, one lap every $9.9 \times 10^3 \text{ s}$ (2.8 h), escape at 8.9 km/s.

Common mistake

Thinking a higher orbit has less energy because the satellite moves more slowly. Moving out to a bigger circle takes energy: K falls, but U_g rises twice as much, so E rises toward zero.

Unit 7 Oscillations

5–8%
of multiple choice

A mass on a spring and a pendulum both swing back and forth in simple harmonic motion. Free-response questions have asked for the graphs of a cart on a spring and for a lab that finds a spring constant with only a stopwatch and a scale.

Not on the exam: damped or driven oscillations, and pendulums swinging through large angles.

7.1 Defining simple harmonic motion

Simple harmonic motion (SHM) happens when the net force pulls an object back toward equilibrium with a size proportional to how far it is displaced. So the acceleration, $a = -(k/m)x$, always points toward equilibrium and is largest at the ends. A mass on a spring moves in SHM, and so, very nearly, does a pendulum on small swings. A bouncing ball repeats, but it is not SHM: in the air, the only force on it is its constant weight.

On the reference sheet $\vec{F}_s = -k\Delta x$

Hanging a mass on a vertical spring only moves the equilibrium down, by mg/k . The motion about the new equilibrium is the same SHM with the same period.

Worked example

A 0.50 kg block hangs on a 50 N/m spring. It is pulled 0.040 m below its resting point and let go. Find its acceleration at the lowest point.

- At rest the stretch is $mg/k = 5.0/50 = 0.10$ m, so at the lowest point it is 0.14 m.
- The spring pulls up with $(50)(0.14) = 7.0$ N and gravity pulls down with 5.0 N: the net force is 2.0 N up.

$$a = 2.0/0.50 = 4.0 \text{ m/s}^2, \text{ up.}$$

Common mistake

Calling the end of a spring oscillator's swing "equilibrium" because the object stops there. Its equilibrium is where the net force is zero, in the middle. At the ends the speed is zero, but the restoring force and the acceleration are at their largest.

7.2 Frequency and period of SHM

The period is the time for one full cycle. A spring's period depends only on the mass and the spring constant, and a pendulum's only on its length and g . Neither depends on the amplitude, and a pendulum's period doesn't depend on the bob's mass.

On the reference sheet $T = \frac{1}{f}$ $T_s = 2\pi\sqrt{\frac{m}{k}}$ $T_p = 2\pi\sqrt{\frac{\ell}{g}}$

Change	Spring's period	Pendulum's period
4 × the mass	× 2	no change
4 × the spring constant	× 1/2	no change
4 × the string length	no change	× 2
2 × the amplitude	no change	no change

In the lab, time ten cycles and divide by ten. T^2 against m is a straight line with slope $4\pi^2/k$: a slope of $1.96 \text{ s}^2/\text{kg}$ gives $k = 20 \text{ N/m}$.

Common mistake

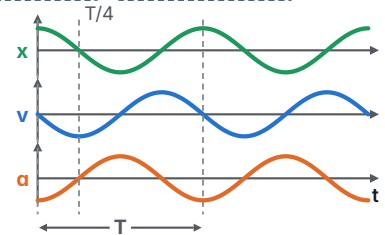
Timing from one end of the swing to the other. That is half a period: a full period ends only when the object is back where it started, moving the same way. If end to end takes 0.80 s, the period is 1.6 s.

7.3 Representing and analyzing SHM

Position is a cosine of time when the object starts at $x = +A$, and a sine when it starts in the middle moving the positive way. Velocity is the slope of the position graph: largest where x crosses zero and zero at the ends. Acceleration is the position graph flipped upside down and scaled. The top speed is $v_{\max} = 2\pi A/T$.

On the reference sheet $x = A \cos(2\pi ft)$ $x = A \sin(2\pi ft)$

The object is released from $x = +A$ at $t = 0$. Where x peaks, $v = 0$ and a is largest in the opposite direction; a quarter period later $x = 0$ and the speed is largest.



Worked example

A cart moves with $x = (0.050 \text{ m}) \cos(2\pi \cdot 2.5 t)$. Find its amplitude, frequency, period and top speed.

- $A = 0.050 \text{ m}$, $f = 2.5 \text{ Hz}$, $T = 0.40 \text{ s}$, $v_{\max} = 2\pi(0.050)/0.40$.

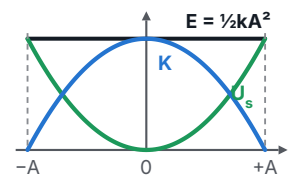
0.050 m, 2.5 Hz, 0.40 s, and 0.79 m/s as it passes $x = 0$.

Common mistake

Calling the distance from one end to the other the amplitude. Amplitude is measured from the middle: a cart swinging between -0.10 m and $+0.10 \text{ m}$ has $A = 0.10 \text{ m}$, not 0.20 m.

7.4 Energy of simple harmonic oscillators

For a horizontal spring without friction, the energy stays constant and moves between the spring and the mass: all U_s at the ends, all K in the middle. Doubling the amplitude gives four times the energy and twice the top speed, with the same period.



On the reference sheet $U_s = \frac{1}{2}k(\Delta x)^2$ $K = \frac{1}{2}mv^2$

Not on the reference sheet $E = \frac{1}{2}kA^2$ $v_{\max} = A\sqrt{k/m}$

Worked example

A 0.50 kg cart on a 200 N/m spring oscillates with amplitude 0.10 m. How fast is it moving at $x = 0.060 \text{ m}$?

- $E = \frac{1}{2}(200)(0.10)^2 = 1.0 \text{ J}$; $U_s = \frac{1}{2}(200)(0.060)^2 = 0.36 \text{ J}$.
- The rest is kinetic: $\frac{1}{2}(0.50)v^2 = 1.0 - 0.36 = 0.64 \text{ J}$.

$v = 1.6 \text{ m/s}$, in either direction.

Common mistake

Assuming the energy is split evenly halfway out. At $x = A/2$ the spring holds only a quarter of it, because U_s grows with x^2 ; the split is even at $x = A/\sqrt{2} \approx 0.71A$.

Unit 8 Fluids

10–15%
of multiple choice

Fluids follow the same laws as everything else. Newton's laws explain pressure and buoyancy, and conservation of mass and energy explain how fluids flow through pipes. Buoyancy and the flow rate of a fountain have both been free-response questions.

Not on the exam: fluids with internal friction (viscosity).

8.1 Internal structure and density

In a solid, each particle is held in place by its neighbors. In a liquid the particles still touch but slide past each other, and in a gas they are far apart. Liquids and gases both flow, so both are fluids. Density is mass per volume: it says how tightly mass is packed, not how heavy something is. Unless stated otherwise, exam fluids are ideal: they can't be squeezed into a smaller volume, and they flow with no internal friction.

On the reference sheet

$$\rho = \frac{m}{V}$$

Not on the reference sheet

$$1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$$

$$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$$

Worked example

A 54 g stone dropped into a graduated cylinder raises the water level from 40.0 mL to 60.0 mL. Find its density.

1 $V = 20.0 \text{ mL} = 20.0 \text{ cm}^3 = 2.00 \times 10^{-5} \text{ m}^3$.

2 $\rho = 0.054 \text{ kg}/2.00 \times 10^{-5} \text{ m}^3$.

$2.7 \times 10^3 \text{ kg/m}^3$, or 2.7 g/cm^3 .

Common mistake

Multiplying cubic centimeters by 10^{-2} to get cubic meters. The stone's 20.0 cm^3 would become 0.200 m^3 and its density 0.27 kg/m^3 , lighter than air. A cubic centimeter is $(10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$.

8.2 Pressure

Pressure is the force pushing straight into a surface divided by the surface's area, measured in pascals (N/m^2). In a fluid at rest, pressure grows with depth h , measured straight down from the surface. P_0 is the pressure on the surface, $1.0 \times 10^5 \text{ Pa}$ for open air, and the gauge pressure ρgh is the part added by the fluid. Only depth matters, not the shape or width of the container. A pressure difference across a surface pushes it with a net force $(P_{\text{high}} - P_{\text{low}})A$.

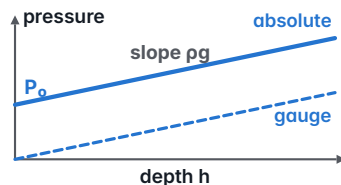
On the reference sheet

$$P = \frac{F_{\perp}}{A}$$

$$P = P_0 + \rho gh$$

$$P_{\text{gauge}} = \rho gh$$

Both lines have slope ρg , so they are steeper in a denser liquid.



Worked example

In an open tank, 0.50 m of oil (800 kg/m^3) floats on 1.00 m of water. Find the gauge pressure at the bottom.

1 Each layer adds its own ρgh : oil $(800)(10)(0.50) = 4.0 \times 10^3 \text{ Pa}$.

2 Water: $(1000)(10)(1.00) = 1.0 \times 10^4 \text{ Pa}$.

$1.4 \times 10^4 \text{ Pa gauge}$, or $1.14 \times 10^5 \text{ Pa absolute}$.

Worked example

A 0.010 m^2 hatch is 2.0 m below the surface of an open water tank, with air on its dry side. Find the net force on it.

1 Gauge pressure: $(1000)(10)(2.0) = 2.0 \times 10^4 \text{ Pa}$.

2 Air pushes on both sides, so only the gauge part counts.

$F = (2.0 \times 10^4)(0.010) = 200 \text{ N}$, outward.

Common mistake

Multiplying the absolute pressure, $1.2 \times 10^5 \text{ Pa}$, by the area gives 1200 N. The answer is 200 N, because air pushes on both sides and its share cancels.

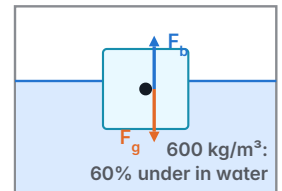
8.3 Fluids and Newton's laws

A fluid pushes harder on the deeper bottom of a submerged object than on its top. The difference is the buoyant force, straight up, equal to the weight of the fluid the object pushes aside. In $F_b = \rho V g$, ρ is the fluid's density and V is the volume under the surface. Fully under, F_b doesn't change with depth.

A floating object sinks until it pushes aside its own weight of fluid, so at rest $F_b = mg$.

On the reference sheet

$$F_b = \rho V g$$



Not on the reference sheet

$$\text{floating: } \frac{V_{\text{under}}}{V_{\text{object}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$$

Worked derivation

A block of mass m and volume V is released from rest fully under a liquid of density ρ . Starting with Newton's second law, derive its acceleration. Up is positive: $F_b - mg = ma$, so $\rho V g - mg = ma$ and $a = \frac{\rho V g}{m} - g$.

Worked example

A 0.60 kg block with a volume of $1.0 \times 10^{-3} \text{ m}^3$ is released fully under fresh water (1000 kg/m^3), then under salt water (1200 kg/m^3). Find each starting acceleration.

1 Fresh water: $F_b = (1000)(1.0 \times 10^{-3})(10) = 10 \text{ N}$ and $mg = 6.0 \text{ N}$, so $a = (10 - 6.0)/0.60 = 6.7 \text{ m/s}^2$ up.

2 Salt water: $F_b = 12 \text{ N}$, so $a = (12 - 6.0)/0.60 = 10 \text{ m/s}^2$ up.

20% denser water gives 50% more acceleration, because the weight is subtracted before dividing by the mass.

Common mistake

Comparing only the buoyant forces. Say that the weight is the same in both liquids, then compare the net forces, $F_b - mg$: above, F_b is 20% larger in salt water but the net force is 50% larger.

8.4 Fluids and conservation laws

Pipes are completely full unless a question says otherwise, and an ideal fluid can't bunch up or thin out, so the same volume passes each section of an unbranched pipe each second: the flow rate Av , in m^3/s , is constant along it. Halving the radius quarters the area and makes the flow four times as fast. For steady flow with no pump between the points, Bernoulli's equation conserves energy per cubic meter along a streamline. At one height, faster flow means lower pressure. Water leaving a small hole into air a depth h below the surface of a wide, open tank comes out at $\sqrt{2gh}$, as fast as if it had fallen h .

On the reference sheet $A_1v_1 = A_2v_2$

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

Not on the reference sheet flow rate = Av from a hole: $v = \sqrt{2gh}$

⚠ Common mistake

Using the diameter as the radius. A pipe 2.0 cm across has $r = 0.010$ m, so $A = \pi(0.010)^2 = 3.1 \times 10^{-4} \text{ m}^2$, not $\pi(0.020)^2 = 1.3 \times 10^{-3} \text{ m}^2$, which is four times too big.

Worked example

Water flows at 2.0 m/s along a level pipe from a section of area 4.0 cm^2 into one of 1.0 cm^2 . Find its new speed and the pressure difference.

- $v_2 = A_1v_1/A_2 = (4.0)(2.0)/1.0 = 8.0 \text{ m/s}$.
- Same height: $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(1000)(64 - 4.0)$.

8.0 m/s; the pressure is $3.0 \times 10^4 \text{ Pa}$ lower in the narrow part.

Worked example

A fountain's jet rises straight up 1.8 m from a nozzle of area 2.0 cm^2 . Find the launch speed and the flow rate.

- The jet is a projectile: at the top $v = 0$, so $0 = v_0^2 - 2gh$ and $v_0 = \sqrt{2(10)(1.8)} = 6.0 \text{ m/s}$.
- Flow rate = $Av = (2.0 \times 10^{-4})(6.0)$.

6.0 m/s and $1.2 \times 10^{-3} \text{ m}^3/\text{s}$, about 1.2 liters a second.

Free response

4 questions
95 minutes, 50%

What earns the points on each free-response question

The four question types come in the same order every year, and any of them can draw on any unit. Each one is scored in small pieces, so a correct starting equation can earn a point even when the final answer is wrong.

1. Mathematical Routines, 10 points

- Start every derivation from a law or a reference-sheet equation in symbols, then substitute. An answer with no starting point can lose the derivation point.
- Finish in the symbols the question allows. An answer that still contains something you were meant to eliminate isn't done.
- Back the claim at the end with physics: say which quantity changes and which law ties it to your claim.

2. Translation Between Representations, 12 points

- Every representation you make has to agree with the others: for example, each force on your diagram needs a matching term in your equation.
- On bar charts, draw a zero as a line on the zero line. On graphs, label each curve and get its shape, slope and zero crossings right.
- To show two answers agree, point to one specific feature (a slope, an intercept, a bar height, a term in the equation) and say what it means in both.

A Question 4 answer, start to finish

The question. Blocks of $m_2 = 3.0 \text{ kg}$ and $m_1 = 2.0 \text{ kg}$ hang over an ideal pulley. A student claims that adding 1.0 kg to each block won't change the acceleration, since the difference in weight stays the same.

Claim. The acceleration decreases: the unbalanced force is unchanged, but it now has to accelerate more total mass.

Derive. For both blocks along the string, $(m_2 - m_1)g = (m_1 + m_2)a$, so $a = (m_2 - m_1)g/(m_1 + m_2)$.

Connect. The numerator stays $(1.0 \text{ kg})g$ while the denominator grows from 5.0 to 7.0 kg, so a drops from 2.0 to 1.4 m/s^2 .

3. Experimental Design and Analysis, 10 points

- Procedure: say what you change, what you keep the same, and what you measure with which instrument. Use several values and repeat each trial.
- Graph: rearrange the physics equation so the graph is a straight line, label both axes with quantities and units, use even scales, and draw one best-fit line.
- Take the slope from two points on your line, far apart, not from data points, and say what physical quantity the slope equals.
- Name a specific source of error and which way it pushes the result. "Human error" earns nothing.

4. Qualitative/Quantitative Translation, 8 points

- Make the claim, then give the chain of reasoning in words.
- Derive the equation, starting from the law the question names.
- Connect them: point to where the changed quantity sits in your equation (in the numerator or the denominator, squared, under a root, or added to something else) and say whether it supports your claim. If it doesn't, say which part of your reasoning was wrong.

When the question says	It wants
Calculate	numbers substituted step by step, and a unit
Derive	symbols, starting from a law or an equation on the reference sheet
Justify	reasons in words, beyond the algebra
Sketch	the right shape and key features, consistent with any supplied axes
Plot	data points placed on labeled, scaled axes
Indicate	just the answer, no explanation needed