

Unit 1

AP Physics 1

Kinematics

Scalars and vectors · Displacement, velocity and acceleration · Motion graphs · Constant-acceleration equations · Reference frames · Projectiles

Unit 1
10–15%
Kinematics

Unit 2
18–23%
Force and dynamics

Unit 3
18–23%
Work, energy, power

Unit 4
10–15%
Linear momentum

Unit 5
10–15%
Torque and rotation

Unit 6 5–8%
Rotating systems

Unit 7 5–8%
Oscillations

Unit 8
10–15%
Fluids

Unit 1 is 10–15% of the multiple-choice section. No free-response question is set aside for kinematics, but its graphs and equations show up inside the others: a recent exam asked for a projectile's velocity graphs in one question and for position graphs through a collision in another.

1.1
Scalars and vectors in one dimension

1.2
Displacement, velocity, and acceleration

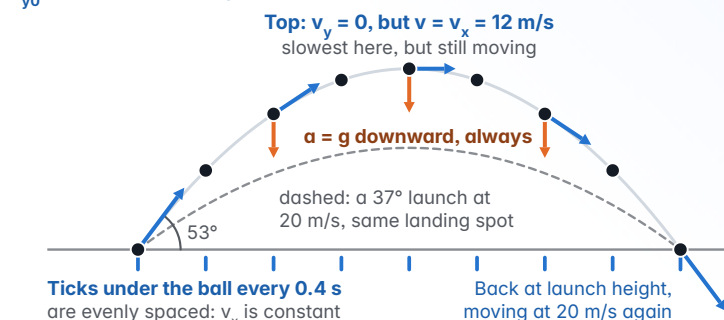
1.3
Representing motion

1.4
Reference frames and relative motion

1.5
Vectors and motion in two dimensions

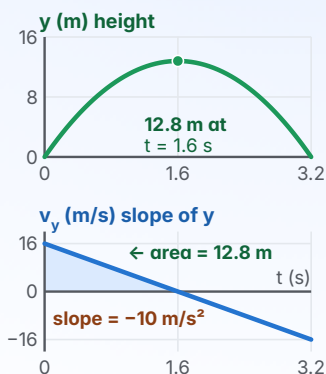
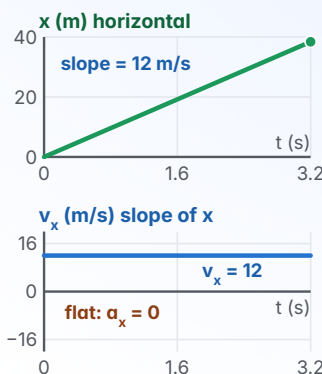
$$v_x = 20 \cos 53^\circ = 12 \text{ m/s}$$

$$v_{y0} = 20 \sin 53^\circ = 16 \text{ m/s}$$



A ball leaves level ground at 20 m/s , 53° above the horizontal ($g = 10 \text{ m/s}^2$, no air resistance). Its sideways and up-and-down motions share the same time: sideways at a steady 12 m/s , and up and down like a ball thrown straight up at 16 m/s .

— position — velocity — acceleration



What the exam asks most

1 Turn one motion graph into another. Given a position graph, draw the velocity graph, or the reverse. The velocity at any moment is the slope of the position graph at that moment, and the displacement over an interval is the area under the velocity graph. **1.3**

2 Draw a projectile's velocity graphs. v_x is a flat line above zero, because nothing pushes the ball sideways. v_y is a straight line that drops by 10 m/s every second and crosses zero at the top of the path. **1.5**

3 Derive an equation, starting from the equation sheet. Write the printed equation first, in its y form: $v_y^2 = v_{y0}^2 + 2a_y\Delta y$. Then substitute, saying why each value is right: $v_y = 0$ at the top, where the ball stops rising, and $v_{y0} = v_0 \sin \theta$. **1.3 1.5**

4 Turn lab data into a straight line. A cart released from rest has $x = \frac{1}{2}at^2$, so a graph of x against t^2 is a straight line with slope $a/2$. Take the slope from two points on your best-fit line, then $a = 2 \times \text{slope}$. **1.3**

5 Tell whether an object speeds up or slows down. It speeds up when v and a have the same sign and slows down when they have opposite signs. With up as positive, a falling ball has $v < 0$ and $a = -10 \text{ m/s}^2$, so it gets faster. **1.1 1.2**

6 Find a velocity seen by a moving observer. Add the velocities along the line, keeping their signs. Walk forward at 1.5 m/s in a train moving at 20 m/s , and someone on the platform sees you pass at 21.5 m/s ; walk toward the back and it is 18.5 m/s . **1.4**

On the equation sheet

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

They hold only while the acceleration is constant, and they work vertically too, with y in place of x . The equation sheet also gives the sine, cosine and tangent of 0° , 30° , 37° , 45° , 53° , 60° and 90° , and $g = 9.8 \text{ m/s}^2$. You may use $g = 10 \text{ m/s}^2$ whenever you need a number, as every example here does; 9.8 or 9.81 also earn credit.

Not on the equation sheet: know these

- Average velocity = $\Delta x / \Delta t$ and average acceleration = $\Delta v / \Delta t$.
- Average speed = distance $\div$ time.
- $\Delta x = \frac{1}{2}(v_{x0} + v_x)t$ when a is constant.
- $v_x = v \cos \theta$ and $v_y = v \sin \theta$, with θ measured from the horizontal.
- $v_{AC} = v_{AB} + v_{BC}$, where v_{AB} is A's velocity measured by B.

Not on the exam

- Calculating with an acceleration that changes; you only sketch and describe that motion.
- Relative velocity in two dimensions, such as a boat crossing a river.

1.1 Scalars and vectors in one dimension

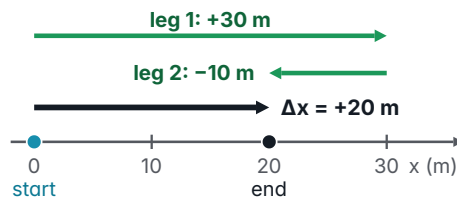
A scalar has only a size. A vector also has a direction, and you draw it as an arrow whose length shows the size.

Scalar: size only **Vector:** size and direction

distance, 40 m displacement, +20 m

speed, 4 m/s velocity, -4 m/s

time, mass position, acceleration



Walk 30 m east, then 10 m west, with east as positive. Displacement adds the signed arrows; distance adds their lengths, 40 m.

A vector's sign is its direction

Pick one direction as positive and keep it for the whole problem. A vector pointing the other way gets a minus sign, so in one dimension you add vectors as signed numbers.

Worked example

The walk in the figure takes 8.0 s. Find its average velocity and its average speed.

- $v_{\text{avg}} = \Delta x / \Delta t = (+20 \text{ m}) / (8.0 \text{ s})$
- speed = distance / $\Delta t = 40 \text{ m} / 8.0 \text{ s}$

Average velocity +2.5 m/s; average speed 5.0 m/s

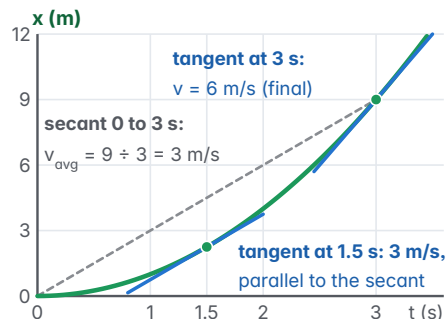
A race with zero average velocity

The 1500 m freestyle, 30 lengths of a 50 m pool, ends at the starting wall, so the displacement and the average velocity are zero. Katie Ledecky's average speed in her Paris Olympic final was $1500 \text{ m} \div 930.02 \text{ s} = 1.61 \text{ m/s}$.

Exam trap

- Reading a negative velocity as slowing down.** The minus sign only gives the direction: a car backing up faster and faster has a negative velocity and is speeding up.
- Writing a negative speed.** Speed is the size of the velocity, so it is never negative: write $v = -5 \text{ m/s}$ or speed = 5 m/s.

1.2 Displacement, velocity, and acceleration



A cart starting from rest with $a = 2 \text{ m/s}^2$ has $x = t^2$. The slope of the dashed secant (a line through two points of the graph) is the average velocity; a tangent's slope is the velocity at one instant. The tangent at 1.5 s is parallel to the secant because, with constant acceleration, the velocity halfway through an interval equals the average. So from rest, the final velocity is **twice** the average.

Four definitions to know by heart

Displacement	$\Delta x = x - x_0$	m
Average velocity	$v_{\text{avg}} = \Delta x / \Delta t$	m/s
Average speed	distance / Δt	m/s
Average acceleration	$a_{\text{avg}} = \Delta v / \Delta t$	m/s ²

None of these is on the equation sheet. An **instantaneous** value is the same ratio over a very short time: the slope of the tangent line. You can treat an object as a point when its size and shape don't matter to the question.

Worked example

A car speeds up steadily from rest. Its positions at $t = 0, 1, 2, 3$ and 4 s are 0, 2, 8, 18 and 32 m. Find its acceleration.

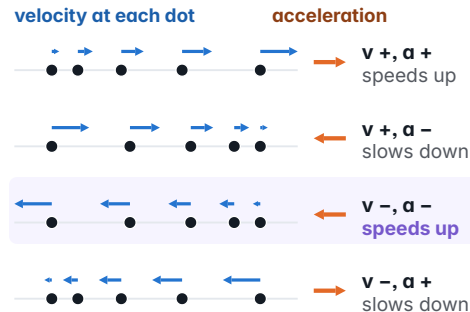
- The average velocity in each 1 s interval is 2, 6, 10 and 14 m/s.
- With constant a , each average is the velocity at the middle of its interval (0.5, 1.5, 2.5, 3.5 s).
- So the velocity rises by 4 m/s every second.

$a = 4.0 \text{ m/s}^2$ (check: $x = \frac{1}{2}(4)t^2 = 2t^2$ gives 0, 2, 8, 18, 32 m)

Exam trap

- Using $\Delta x / \Delta t$ as the final velocity.** $\Delta x / \Delta t$ is the average velocity for the whole trip, and a cart that is speeding up finishes faster than its average. From rest with constant acceleration, the final velocity is twice the average: a cart that rolls 1.2 m from rest in 2.0 s averages 0.60 m/s and finishes at 1.2 m/s.

velocity at each dot



Dots are equal times apart. An object speeds up when v and a point the same way and slows down when they point opposite ways, so in the shaded row it speeds up even though a is negative.

Worked example

A ball hits the floor at 6.0 m/s and leaves at 4.0 m/s after 0.050 s of contact. Taking up as positive, find its average acceleration during the contact.

- $v_{y0} = -6.0 \text{ m/s}$ (moving down), $v_y = +4.0 \text{ m/s}$
- $\Delta v = (+4.0) - (-6.0) = +10.0 \text{ m/s}$
- $a_{\text{avg}} = 10.0 / 0.050$

$+2.0 \times 10^2 \text{ m/s}^2$, upward. Because the ball reverses direction, Δv is $6.0 + 4.0 = 10 \text{ m/s}$.

Acceleration means the velocity is changing

An object accelerates whenever its velocity changes in size or in direction. A car rounding a bend at a steady 15 m/s is accelerating. A ball thrown straight up has $v = 0$ at the top but $a = -g$: zero velocity doesn't mean zero acceleration.

Worked example

A car drives 1200 m at 20 m/s, then 1200 m at 30 m/s. What is its average speed for the whole trip?

- Times: $1200 / 20 = 60 \text{ s}$ and $1200 / 30 = 40 \text{ s}$.
- Total distance $\div$ total time: $2400 \text{ m} \div 100 \text{ s}$.

24 m/s. That is less than 25 m/s, the mean of the two speeds, because the car spends longer at the slower speed.

1.3 Representing motion with graphs

Velocity is the slope of position;
acceleration is the slope of velocity

The motion	Motion diagram	Position x	slope $\rightarrow$ Velocity v	slope $\rightarrow$ Acceleration a
At rest x stays the same	● one dot, no arrow			
Constant velocity, + x rises in a straight line				
Speeding up, moving + v and a have the same sign				
Slowing down, moving + v and a have opposite signs				
Goes out, stops, comes back at the turn $v = 0$, but $a \neq 0$				
Speeding up, moving - $a < 0$, but it speeds up				

In the green row the object goes out along the upper row of dots, stops, and comes back along the lower, grey row. The dashed line marks the turnaround on all three graphs: the acceleration stays negative straight through it.

— position — velocity — acceleration

What the slope and the area mean

Graph	Slope	Area under it
x vs t	velocity	not used
v vs t	acceleration	displacement
a vs t	not used	change in velocity
speed vs t	not used	distance

Area below the time axis counts as negative. Under a straight piece of a v - t graph the area is a trapezoid, which gives the equation $\Delta x = \frac{1}{2}(v_{x0} + v_x)t$.

Reading a position graph

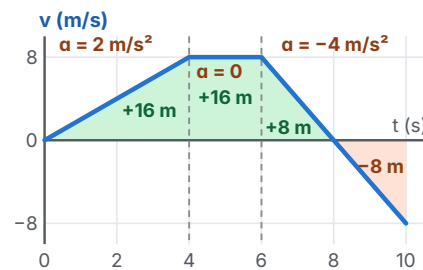
- A steeper line means the object is moving faster. A flat line means it is at rest.
- Bending upward (like a smile) means $a > 0$, bending downward means $a < 0$, and a straight line means $a = 0$.
- A peak or a valley is where the object turns around, so $v = 0$ there.
- Crossing the time axis just means passing $x = 0$.
- For the velocity at one moment, draw the tangent there (a straight line that just touches the curve) and take its rise over run between two far-apart points.

Exam trap

- Reading a graph as a picture of the path. A v - t graph shaped like a hill doesn't mean the object went over a hill. It means the object sped up and then slowed down.

Worked example

A cart starts at $x = 0$. Find each acceleration, the displacement and the distance traveled in 10 s, and when the cart is farthest from its start.



- 1 The slopes are the accelerations: 2 m/s^2 , 0 , then -4 m/s^2 .
- 2 The areas are the displacements: $+16$, $+16$ and $+8 \text{ m}$ above the axis, -8 m below it.
- 3 Displacement adds the signed areas: 32 m . Distance adds their sizes: 48 m .
- 4 The cart moves forward until $v = 0$ at 8 s , so it is farthest away then, 40 m from the start.

$\Delta x = +32 \text{ m}$; distance 48 m ; farthest at 8 s

Quick check

A v - t graph runs in a straight line from $+6 \text{ m/s}$ at $t = 0$ to -6 m/s at $t = 4 \text{ s}$. When is the object farthest from its start, and what is its displacement at 4 s ?

Answer: at 2 s , when $v = 0$. The triangles above and below the axis have equal areas, so $\Delta x = 0$: it is back where it started.

Reading an acceleration graph

If a stays at $+3 \text{ m/s}^2$ for 4 s , the area under the a - t graph, $3 \times 4 = 12 \text{ m/s}$, is the change in velocity. For the velocity itself you also need the starting velocity: a cart that starts at 5 m/s ends at 17 m/s .

When the acceleration changes



The v - t graph curves, and the slope of the tangent at a moment is the acceleration at that moment. The exam asks you to sketch and describe these motions, never to calculate with them.

Exam trap

- Reading the height instead of the slope. A high point on an x - t graph means the object is far out in the $+$ direction, not that it is fast. At the top of a peak it is stopped for an instant.
- Reading a crossing as equal speeds. Where two lines cross on an x - t graph, the two objects are at the same place. They have the same velocity only where the lines are parallel.

1.3 The constant-acceleration equations and free fall The equations work only while the acceleration stays constant

Choosing the equation

Equation	Δx	v_{x0}	v_x	a_x	t	Leaves out	On the equation sheet?
$v_x = v_{x0} + a_x t$	—	•	•	•	•	displacement	yes
$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	•	•	—	•	•	final velocity	yes
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	•	•	•	•	—	time	yes
$x - x_0 = \frac{1}{2}(v_{x0} + v_x)t$	•	•	•	—	•	acceleration	no: you get it from the area under a straight v - t line

A dot means the equation contains that quantity; a dash marks the one it leaves out. A problem gives you three of the five quantities and asks for a fourth; use the equation that leaves out the fifth, the one you neither know nor need. The equation sheet writes x , but the same equations work for vertical motion with y in place of x . Remember that $x - x_0$ is a displacement, so it can be negative.

Worked example

A car at 12 m/s brakes at a steady 3.0 m/s² until it stops. How long does it take, and how far does it go?

- Forward is positive, so $v_{x0} = 12$ m/s, $v_x = 0$ and $a_x = -3.0$ m/s² (the car slows).
- For the time: $0 = 12 + (-3.0)t$, so $t = 4.0$ s.
- For the distance, the equation without t (so it doesn't depend on step 2): $0 = 12^2 + 2(-3.0)\Delta x$, so $\Delta x = 24$ m.
- Check with the area under the v - t line: $\frac{1}{2}(4.0 \text{ s})(12 \text{ m/s}) = 24$ m.

4.0 s and 24 m

Worked example

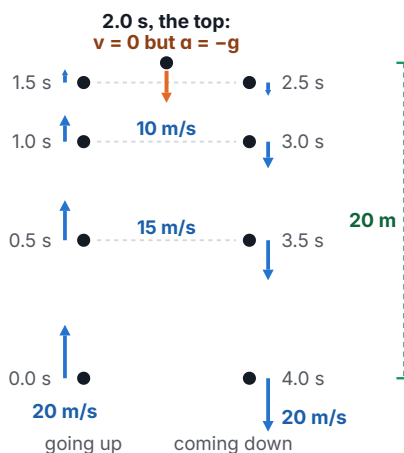
The same car brakes the same way from 24 m/s. How far does it go before it stops?

- With $v_x = 0$ and $a_x = -3.0$ m/s², $v_x^2 = v_{x0}^2 + 2a_x\Delta x$ gives $\Delta x = v_{x0}^2/(6.0 \text{ m/s}^2)$, so doubling the speed quadruples the distance.

4 × 24 m = 96 m

Exam trap

- Using one equation across a change in acceleration. A toy rocket that speeds up while its engine burns, then coasts, needs two intervals. If it rises from rest at +20 m/s² for 2.0 s, the burn ends at $v = 40$ m/s and $y = 40$ m, and the coast starts from those values with $a = -10$ m/s².



A ball thrown up at 20 m/s, shown every 0.5 s. At each height its speed coming down matches its speed going up.

— velocity — acceleration

Free fall

- With up as positive, $a_y = -g$ the whole time, even at the top.
- Without air resistance, rising to the top takes as long as falling back to the launch height.
- You may use $g = 10$ m/s² in calculations, as the exam does; 9.8 (the equation sheet's value) or 9.81 also earn credit.

Worked example

A rock dropped from a cliff lands 3.0 s later. How high is the cliff, and how fast is the rock moving when it lands?

- Take up as positive: $v_{y0} = 0$, $a_y = -10$ m/s², $t = 3.0$ s.
- $\Delta y = \frac{1}{2}(-10)(3.0)^2 = -45$ m, so it ends 45 m below its start.
- $v_y = 0 + (-10)(3.0) = -30$ m/s

The cliff is 45 m high, and the rock lands at 30 m/s.

A derivation that starts from the equation sheet

Show that a ball thrown straight up at v_0 rises to a height $h = v_0^2/(2g)$.

$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ (from the equation sheet)

At the top the ball stops rising, so $v_y = 0$. Also $v_{y0} = v_0$, $a_y = -g$ and $y - y_0 = h$.

$0 = v_0^2 - 2gh$, so $h = v_0^2/(2g)$.

Start from the printed equation, say why each value goes in, and only then solve. With the result you can reason: doubling v_0 makes h four times as large.

Exam trap

- Losing the sign of g . With up as positive, $a_y = -10$ m/s² for the whole flight, and a ball that lands below where it was thrown has a negative Δy . A ball thrown up at 10 m/s from a 15 m roof lands with $\Delta y = -15$ m; using +15 m makes the equation have no real solution.
- Thinking the acceleration is zero at the top. The velocity is zero there for an instant, but gravity still pulls, so $a = -g$. If a were zero, the ball would stay up.

Worked example

That ball, thrown up at 10 m/s from the edge of a 15 m roof, misses the roof on the way down. When does it hit the ground?

- Up positive: $-15 = 10t + \frac{1}{2}(-10)t^2$, so $5t^2 - 10t - 15 = 0$.
- Divide by 5: $t^2 - 2t - 3 = 0$, which factors as $(t - 3)(t + 1) = 0$.
- Time can't be negative, so keep $t = 3$ s.

$t = 3.0$ s, and it lands at $10 - 10(3.0) = -20$ m/s, 20 m/s downward.

The hammer and the feather

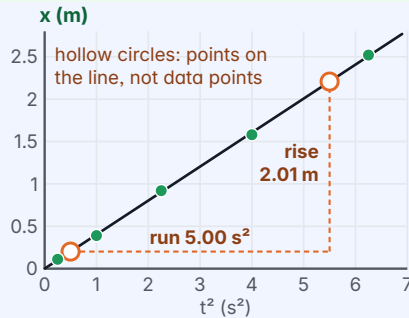
On 2 August 1971, Apollo 15 astronaut David Scott dropped a hammer and a falcon feather on the Moon. With no air to slow the feather, they hit the ground together: free-fall acceleration doesn't depend on mass.

1.3 Graphs from lab data and from a description

Finding the acceleration from lab data

A cart is released from rest down a ramp.

t (s)	0.50	1.00	1.50	2.00	2.50
x (m)	0.11	0.39	0.92	1.58	2.52
t^2 (s ²)	0.25	1.00	2.25	4.00	6.25

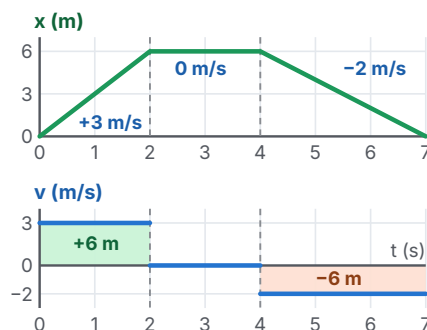


- Start from the equation sheet: $x = x_0 + v_{x0}t + \frac{1}{2}at^2$. Starting at $x_0 = 0$ from rest, $x = \frac{1}{2}at^2$.
- So x plotted against t^2 is a straight line with slope $a/2$.
- Plot on evenly spaced scales, label both axes with units, and draw one straight best-fit line. Don't join the dots, and let the data decide whether the line passes through the origin.
- Take the slope from two far-apart points on your line, not from data points. Here (0.50 s², 0.20 m) and (5.50 s², 2.21 m) give $2.01 \text{ m} \div 5.00 \text{ s}^2 = 0.40 \text{ m/s}^2$.

$$a = 2 \times \text{slope} = 0.80 \text{ m/s}^2$$

Worked example

A walker's position graph is below. Draw the velocity graph.



- Each straight piece has one slope, so the velocity graph is flat in each piece.
- The slopes are $6/2 = +3 \text{ m/s}$, then 0, then $-6/3 = -2 \text{ m/s}$.
- Check: the areas $+6 \text{ m}$ and -6 m cancel, so the walker ends at the start.

Exam trap

- Drawing a straight line through curved data.** For a cart that is speeding up, x against t curves upward, so a straight best-fit line through it earns no credit. Plot x against t^2 , which is straight.
- Blaming "human error".** It earns nothing. Name a specific cause and say which way it pushes the result. Starting the timer late makes every t too small, and since $a = 2x/t^2$, the value of a comes out too large.

Checking the result against the ramp

Ignoring friction and the spinning of the wheels, a cart on a ramp tilted at θ above the floor speeds up at $a = g \sin \theta$ (Unit 2 shows why). For a 3.00 m ramp with one end raised 0.24 m, $\sin \theta = 0.24/3.00 = 0.080$, so $a = 10 \times 0.080 = 0.80 \text{ m/s}^2$, which matches the data. Use the sine, not the cosine, when θ is measured from the floor. If your measured a comes out low, name a cause, such as friction at the axles.

What earns the points on a sketched graph

- Start each graph at the value the question gives (x_0 or v_0) and end it at the time asked.
- Get the sign of the slope right in every interval, and then the way the graph curves. A v - t graph is a straight line wherever a is constant.
- Line up the same moments across your graphs: where the x - t graph is flat, the v - t graph is at zero.
- Draw position as one unbroken line, since an object can't jump from place to place.

Quick check

A cart starts from rest. A graph of x/t against t is a straight line with slope 0.60 m/s^2 . What is a ?

Answer: $x = \frac{1}{2}at^2$, so $x/t = \frac{1}{2}at$ and the slope is $a/2$: $a = 2 \times 0.60 = 1.2 \text{ m/s}^2$.

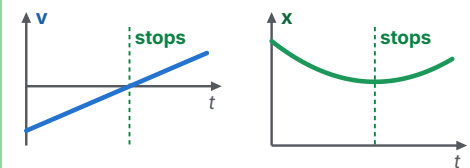
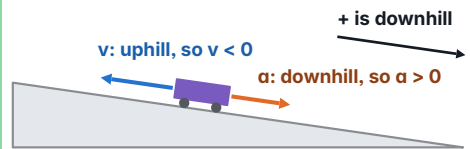
Quick check

An x - t graph is a straight line from 12 m at $t = 0$ down to 0 m at $t = 4$ s. Describe the v - t graph.

Answer: a flat line at -3 m/s from 0 to 4 s, since the slope is $(0 - 12)/4$.

Worked example

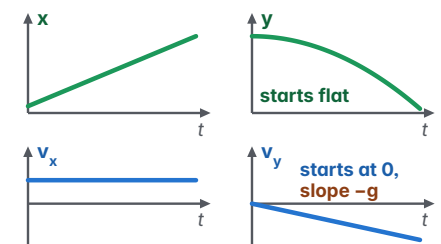
A cart is given a push up a ramp. It slows down, stops for an instant, then rolls back down. Downhill is the positive direction. Sketch its velocity and position graphs from just after the push.



- While the cart goes uphill, it moves in the negative direction, so its velocity starts negative.
- Gravity pulls it downhill the whole time, so a is positive and constant, even while it slows down. The v - t graph is a straight line with positive slope that crosses zero when the cart stops.
- The x - t slope is the velocity, so the graph slopes down, goes flat when the cart stops, then slopes up. Since $a > 0$, it curves upward throughout.

v rises in a straight line through zero; x falls, levels off and rises, curving upward.

Graphs for a ball launched level from a height



v_x stays at the launch speed the whole time, while v_y grows downward by 10 m/s every second. **1.5**

How to show the two ramp graphs agree

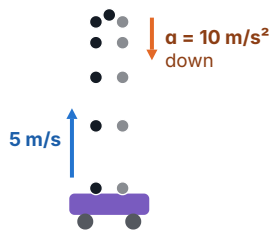
Write a sentence like this one:

At the moment the cart stops, my position graph is flat, so its slope, the velocity, is zero. My velocity graph crosses zero at that same moment, so the two graphs are consistent.

1.4 Reference frames and relative motion

Seen by the rider

straight up, straight down

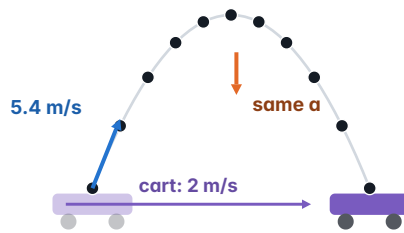


0.1 s between dots; up dark, down light

A rider on a cart rolling at 2 m/s tosses a ball straight up at 5 m/s. From the ground, the ball also keeps the cart's 2 m/s sideways, so it leaves at $\sqrt{2^2 + 5^2} = 5.4$ m/s and follows a parabola.

Seen from the ground

a parabola; $v_x = 2$ m/s throughout



lands back in the hand

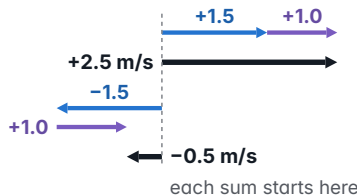
Changing observers (not on the equation sheet)

$$V_{AC} = V_{AB} + V_{BC}$$

- V_{AB} means the velocity of A measured by B. When you add, the two inner letters match (B and B) and drop out, leaving V_{AC} .
- Swapping the letters flips the sign: $V_{BA} = -V_{AB}$.
- Use it along one line only, keeping every sign.

Worked example

You walk at 1.5 m/s on a moving walkway that runs at 1.0 m/s. What is your velocity relative to the ground when you walk forward, and when you walk back?



each sum starts here

- you, relative to the belt — the belt
- you, seen from the ground

- $V_{\text{you,ground}} = V_{\text{you,belt}} + V_{\text{belt,ground}}$
- Forward: $1.5 + 1.0$. Back: $-1.5 + 1.0$.

+2.5 m/s forward; -0.5 m/s walking back, so you drift slowly backward.

Exam trap

- Subtracting in the wrong order.** The velocity of B measured by A is $V_{BA} = V_B - V_A$, with both measured from the ground. A truck ahead of you at 20 m/s, seen from your car at 25 m/s, moves at $20 - 25 = -5$ m/s, which means you are closing in on it.

Worked example

You drive east at 25 m/s. Car P ahead goes east at 30 m/s; car Q comes toward you, west at 25 m/s. With east positive, what velocity do you measure for each?

- From your car the road moves west: $V_{\text{road,you}} = -25$ m/s.
- $V_{P,\text{you}} = V_{P,\text{road}} + V_{\text{road,you}} = 30 + (-25)$
- $V_{Q,\text{you}} = -25 + (-25)$

P: +5 m/s, pulling away. Q: -50 m/s, closing at 50 m/s.

Inertial frames

An inertial frame is one that isn't accelerating: a train at steady speed on straight track is one, a car speeding up or turning is not. Exam questions use inertial frames unless they say otherwise. Every inertial observer measures the same acceleration for an object, even when their velocities differ.

Ground speed and airspeed

In February 2019 a Virgin Atlantic jet riding a jet stream of over 200 mph reached about 800 mph over the ground, faster than sound at that height (about 660 mph), yet made no sonic boom. A boom depends on a plane's speed through the air, and the jet stream carried the air along with the plane.

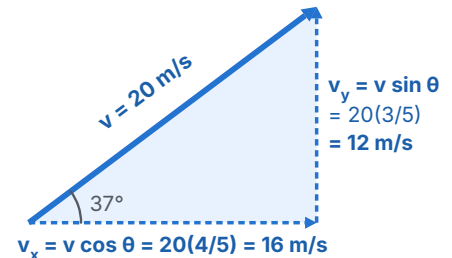
Quick check

A plane flies 600 km out and back at 200 m/s relative to the air, with a 50 m/s wind along the route. How does its total time compare with a calm day?

Answer: $600\,000/250 = 2400$ s out and $600\,000/150 = 4000$ s back: 6400 s, against 6000 s on a calm day. The plane spends longer in the headwind, so the wind adds time.

1.5 Vectors and motion in two dimensions

Any vector splits into two perpendicular components; work out each direction on its own.



Components (not on the equation sheet)

$$v_x = v \cos \theta, \quad v_y = v \sin \theta$$

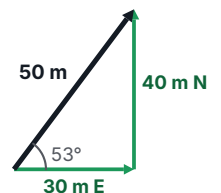
$$v = \sqrt{v_x^2 + v_y^2}, \quad \tan \theta = v_y/v_x$$

θ is measured from the +x axis (37° in the figure). The equation sheet lists trig values like these, but not these vector forms.

θ	30°	37°	45°	53°	60°
$\sin \theta$	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$
$\cos \theta$	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2

Worked example

Walk 30 m east, then 40 m north. Find your displacement and the distance walked.



- Size: $\sqrt{30^2 + 40^2} = 50$ m
- Direction: $\tan \theta = 40/30$, so $\theta = 53^\circ$ north of east.
- Distance adds the legs: $30 + 40 = 70$ m.

Displacement 50 m at 53° north of east; distance 70 m

Exam trap

- Using $\cos$ for v_x when θ is measured from the vertical. The cosine goes with the side next to the angle, so for a ball thrown 30° from the vertical, $v_x = v \sin 30^\circ$ and $v_y = v \cos 30^\circ$.

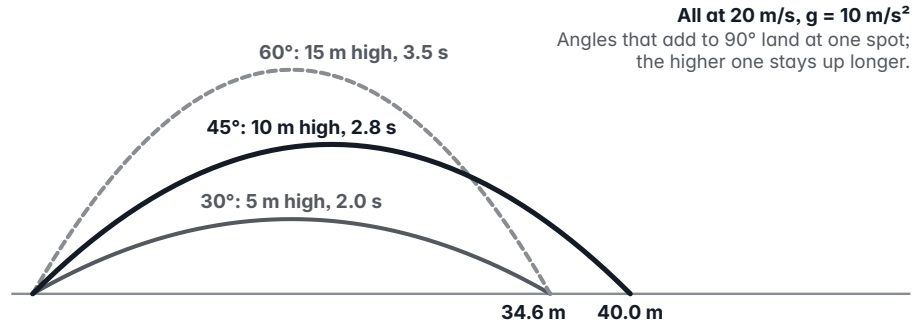
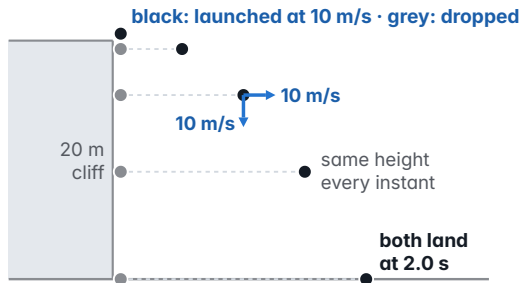
Quick check

Find the components of 10 m/s at 30° above the horizontal.

Answer: $v_x = 10 \cos 30^\circ = 8.7$ m/s; $v_y = 10 \sin 30^\circ = 5.0$ m/s.

1.5 Projectile motion

The horizontal velocity stays constant;
the vertical motion is free fall



Left: a ball dropped and a ball launched sideways from the same height (dots 0.5 s apart) land at the same time, because moving sideways doesn't change how fast a ball falls. **Right:** the same launch speed at three angles, landing at the height they started from, with no air resistance. Each label gives the greatest height and the time in the air; 45° goes farthest.

The two motions

	Horizontal	Vertical (up +)
a	0	$-g$
v	$v_x = v_0 \cos \theta$, constant	$v_y = v_{y0} - gt$
position	$x = x_0 + v_x t$	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$

Each entry comes from a printed equation, with $v_x = v_0 \cos \theta$ and $v_{y0} = v_0 \sin \theta$. The two directions share only the time.

Shortcuts you can derive (not on the equation sheet)

Time to the top	v_{y0}/g
Rise above the launch point	$v_{y0}^2/(2g)$
Time in the air	$2v_{y0}/g$
Range	$v_0^2 \sin 2\theta/g$

The first two work for any upward launch. The last two work only when the ball lands at the height it was launched from; $\sin 2\theta$ is the sine of twice the launch angle. On a derivation, show the steps instead of quoting these.

What each result depends on

Time in the air	vertical motion only
Range	$v_x \times$ time in the air
Doubling v_0 (same angle, equal heights)	doubles the time and makes the height and the range 4 times as big

Writing a justified comparison

Balls A and B roll off the same table, and B is moving faster. Which lands farther from the table?

Both balls start with $v_{y0} = 0$ and fall the same height with the same acceleration g , so they are in the air for the same time. B has the larger v_x , so it covers more ground in that time.

Worked example

A ball rolls off a 1.25 m high table at 3.0 m/s. Where does it land, and how fast is it moving then?

- Vertical first, since it sets the time. Taking down as positive for this fall, $1.25 = \frac{1}{2}(10)t^2$, so $t = 0.50 \text{ s}$.
- Horizontal: $x = (3.0)(0.50) = 1.5 \text{ m}$.
- At landing $v_y = gt = 5.0 \text{ m/s}$ downward, and v_x is still 3.0 m/s.
- Speed = $\sqrt{3.0^2 + 5.0^2}$

It lands 1.5 m from the table, moving at 5.8 m/s.

Deriving a launch speed from the height reached

A water jet leaves a nozzle at angle θ above the horizontal and rises h above it. Find its launch speed.

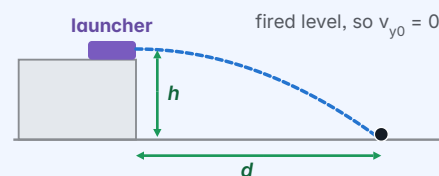
$$v_y^2 = v_{y0}^2 + 2a_y(y - y_0) \text{ (from the equation sheet)}$$

At the top the water stops rising, so $v_y = 0$. The vertical start is $v_{y0} = v_0 \sin \theta$, and $a_y = -g$.

$$0 = v_0^2 \sin^2 \theta - 2gh, \text{ so } v_0 = \frac{\sqrt{2gh}}{\sin \theta}$$

Saying why $v_y = 0$ at the top earns a point, and so does using $v_0 \sin \theta$ rather than v_0 .

Measuring a launcher's speed in a lab



Fire it level from height h and measure the landing distance d . The fall takes $t = \sqrt{2h/g}$, so $v_0 = d/t$. With $h = 0.80 \text{ m}$ and $d = 2.4 \text{ m}$, $t = 0.40 \text{ s}$ and $v_0 = 6.0 \text{ m/s}$. Average d over several shots to cut random error.

Exam trap

- Saying the speed is zero at the top.** Only v_y is zero there; the ball still moves sideways at v_x .
- Putting v_0 into a vertical equation.** The vertical motion starts at $v_0 \sin \theta$: a ball launched at 20 m/s and 37° rises as if thrown straight up at 12 m/s.
- Using the range shortcut off a cliff.** $v_0^2 \sin 2\theta/g$ works only if the ball lands at its launch height. Otherwise find the time from the vertical motion, then multiply by v_x .
- Leaving the calculator in radian mode.** Then $\sin 30$ gives -0.988 , not 0.5.

Set degree mode

TI-84 Plus CE. [mode], arrow down to the row Radian DEGREE, highlight DEGREE, press [enter], then [2nd] [mode] to quit.

TI-Nspire CX. [on] for the home screen, then 5 Settings, 2 Document Settings. Set Angle to Degree, move to Make Default and press [enter].

Desmos. Open the wrench (graph settings) and choose Degrees.

Quick check

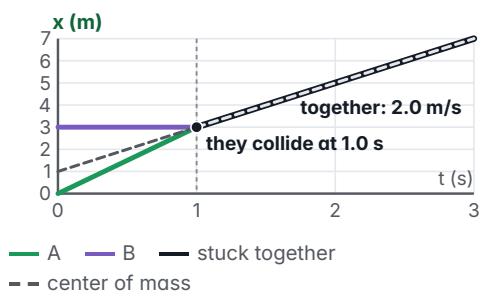
A plane flying level at 50 m/s drops a package from 500 m. How long does it fall, and where does it land compared with the plane?

Answer: $t = \sqrt{2(500)/10} = 10 \text{ s}$. It keeps the plane's 50 m/s, so it lands 500 m ahead of the drop point, right under the plane.

What earns points on the free-response questions

What the question asks, and what earns the point

The question asks	What earns the point
Sketch v_x and v_y against t for a projectile (1.5)	Draw v_x as a flat line above zero. Draw v_y as one straight line sloping down that crosses zero at the top of the path, and stop both at the landing time.
Derive the launch speed from the height reached (1.3) (1.5)	Write $v_y^2 = v_{y0}^2 + 2a_y\Delta y$ first. Put in $v_{y0} = v_0 \sin \theta$ and $v_y = 0$ at the top, say why v_y is zero there, then solve using only the symbols the question gives.
What would you plot to get a straight line? (1.3)	Start from the equation sheet: from rest, $x = \frac{1}{2}at^2$. Say you would plot x against t^2 , and say how the slope gives a : $a = 2 \times \text{slope}$.
Plot the data and draw a best-fit line (1.3)	Use evenly spaced scales that fill most of the grid, label both axes with units, plot each point carefully, and draw one straight line that follows the trend. Don't force it through every point or through the origin.
Use your graph to find a quantity (1.3)	Pick two far-apart points on your line, not data points. Show rise over run with units, then the step that turns the slope into the answer, such as $a = 2 \times \text{slope}$.
Sketch x and v for a cart that is pushed up a ramp and rolls back down (1.2) (1.3)	Use the positive direction the question states. With downhill positive, v starts negative and rises in a straight line through zero; x falls, levels off where $v = 0$, then rises, curving upward the whole time.
Are your answers consistent? (1.3)	Name one feature that appears in both answers (a slope, an intercept, a zero crossing or an area), give its value in each, and say whether they agree.
Greater than, less than or equal to? Justify. (1.5)	State your answer, then the physics in words: which motion matters (the vertical motion sets the time in the air), what is the same and what is different. Equations alone, with no words, don't earn the point.
Why is the measured value too small or too large? (1.3)	Name a specific cause and the way it pushes the result: friction lowers a ; starting the timer late makes t too small and a too large. "Human error" earns nothing.
Position graphs through a collision (with Unit 4) (1.3)	Keep each line unbroken at the collision and change its slope to the new velocity. The center of mass (see the example below) keeps one slope if no outside force acts.



Five checks for every answer

- 1 The positive direction is marked, and every sign follows it.
- 2 Every number has its unit.
- 3 a is constant wherever you used a constant-acceleration equation.
- 4 A projectile's x and y motions are worked separately, sharing one t .
- 5 Your graphs agree with each other.

Worked example

Cart A (2.0 kg) rolls at 3.0 m/s toward cart B (1.0 kg), at rest 3.0 m ahead. They stick together. Sketch both position graphs and the center of mass. (This uses momentum from Unit 4.)

- 1 Before: A's line has slope 3.0 m/s; B's line is flat at 3.0 m. A reaches B at $t = 1.0$ s.
- 2 Momentum is conserved (Unit 4): $(2.0 \text{ kg})(3.0 \text{ m/s}) = (3.0 \text{ kg})v$, so $v = 2.0$ m/s after.
- 3 After: one line of slope 2.0 m/s, starting where both lines meet, with no jump.
- 4 The center of mass, $x_{cm} = (2.0x_A + 1.0x_B)/3.0$, is the mass-weighted average position. With no outside horizontal force, its line keeps a steady slope of 2.0 m/s.

The four free-response questions

Mathematical Routines 10 points

You derive an equation and calculate with it, use a diagram or graph, and justify a claim.

Translation Between Representations 12 points

You draw a diagram, write equations and sketch a graph of one situation, then justify that they agree or use them to make a prediction.

Experimental Design and Analysis 10 points

You describe a procedure, then analyze real data, usually by making a straight-line graph.

Qualitative/Quantitative Translation 8 points

You make a claim and justify it in words, derive an equation, and finally check that your claim and equation agree, or use them to predict.

What the task words ask for

Calculate	a number with its unit, with the steps shown
Derive	start from a law or a printed equation, then do the algebra
Sketch	the right shape, with zeros, peaks and turns in the right places
Plot	data points placed on a labeled scale
Justify	reasons in words, beyond the math
Indicate	just state the answer, no explanation needed

Where Unit 1 comes back

Unit 2	Forces give the acceleration, $a = F_{\text{net}}/m$, that these equations use.
Unit 3	Energy gives speeds without needing the time.
Unit 4	Position graphs through collisions, as in the example.
Units 5–6	Spinning objects have matching equations, such as $\omega = \omega_0 + \alpha t$.
Unit 7	An oscillator's velocity is still the slope of its x - t graph.
Unit 8	Water leaving a hole in a tank moves as a projectile.