

Unit 2

AP Physics 1

Force and Translational Dynamics

Systems · Center of mass · Free-body diagrams · Newton's three laws · Gravitation · Friction · Springs · Circular motion

Unit 1
10–15%
Kinematics

Unit 2
18–23%
Force and dynamics

Unit 3
18–23%
Work, energy, power

Unit 4
10–15%
Linear momentum

Unit 5
10–15%
Torque and rotation

Unit 6 5–8%
Rotating systems

Unit 7 5–8%
Oscillations

Unit 8
10–15%
Fluids

Unit 2 is 18–23% of the multiple-choice section. Any of the four free-response questions can be about forces: a free-body diagram to draw, an equation to derive from Newton's second law, or a lab such as measuring a coefficient of friction.

2.1
Systems and center of mass

2.2
Forces and free-body diagrams

2.3
Newton's third law

2.4
Newton's first law

2.5
Newton's second law

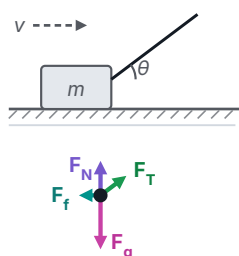
2.6
Gravitational force

2.7
Kinetic and static friction

2.8
Spring forces

2.9
Circular motion

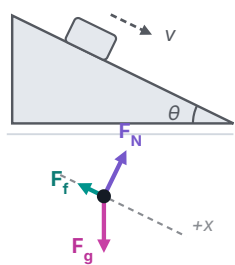
2.7 Dragged across a rough floor



$$F_T \cos \theta - F_f = ma$$

$$F_N = mg - F_T \sin \theta$$

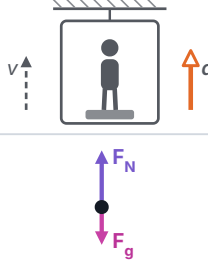
2.5 Sliding down a ramp



$$mg \sin \theta - F_f = ma$$

$$F_N = mg \cos \theta$$

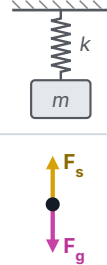
2.6 Elevator speeding up on the way up



$$F_N - mg = ma$$

A scale under the rider reads more than mg .

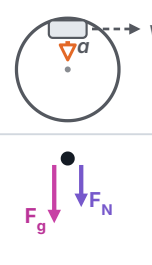
2.8 Hanging at rest on a spring



$$k\Delta x - mg = 0$$

so $\Delta x = mg/k$

2.9 Top of a loop



$$mg + F_N = \frac{mv^2}{r}$$

Both forces point to the center.

Most force problems are solved the same way. Choose the object, draw only the forces exerted on it (each one by something you can name), take the positive direction along the acceleration, and write Newton's second law, $\Sigma F = ma$, in that direction.

$\rightarrow F_g$ gravity $\rightarrow F_N$ normal $\rightarrow F_T$ tension $\rightarrow F_f$ friction $\rightarrow F_s$ spring $\rightarrow F$ a push or pull $\rightarrow v$ velocity $\rightarrow a$ acceleration (not a force)

What the exam asks most

1 A free-body diagram. Draw a dot for the object and one labeled arrow for each force, starting at the dot and pointing the way the force acts. Arrows that point the same way go side by side, and velocity, which isn't a force, never gets an arrow. **(2.2)**

2 A derivation from Newton's second law. Write $\Sigma F = ma$ for one object in one direction, give every force a plus or minus sign, and solve using only the symbols the question gives you. **(2.5)**

3 A normal force that isn't mg . On a ramp, under a rope pulling up at an angle, or in an accelerating elevator, F_N is not mg . Add up the forces perpendicular to the surface and set them equal to ma in that direction. **(2.2) (2.6)**

4 Circular motion. The net force toward the center (real forces pointing in, minus any pointing out) must equal mv^2/r . There is no separate "centripetal force" arrow. **(2.9)**

5 Whether an object slides. Find the friction needed to hold it still and compare it with the most static friction can give, $\mu_s F_N$. If less is needed, the object stays put. **(2.7)**

6 Forces between two objects. They are equal and opposite, even when one object is far heavier. If nothing else pushes much on them, the one with less mass gets the bigger acceleration. **(2.3)**

Formulas for this unit

Given on the equation sheet

Second law	$\vec{a}_{\text{sys}} = \frac{\Sigma \vec{F}}{m_{\text{sys}}}$
Center of mass	$\vec{x}_{\text{cm}} = \frac{\Sigma m_i \vec{x}_i}{\Sigma m_i}$
Gravity	$ \vec{F}_g = G \frac{m_1 m_2}{r^2}$
Friction	$ \vec{F}_f \leq \mu \vec{F}_N $
Spring	$\vec{F}_s = -k\Delta \vec{x}$
Circles	$a_c = \frac{v^2}{r}, T = \frac{1}{f}$

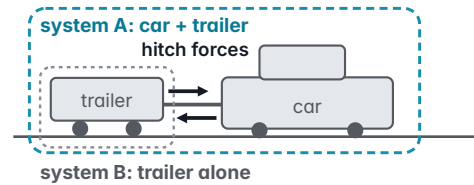
Not given: know these by heart

Third law	$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$
Weight	$F_g = mg$
Sliding	$F_f = \mu_k F_N$
Not sliding	$F_f \leq \mu_s F_N$
Speed in a circle	$v = \frac{2\pi r}{T}$

The sheet also lists $G, g = 9.8 \text{ m/s}^2$, and sines and cosines of common angles: $\sin 37^\circ = 0.60, \cos 37^\circ = 0.80$. Questions use $g = 10 \text{ m/s}^2$ to keep the arithmetic simple, and 9.8 or 9.81 also earn full credit. The examples here use 10.

2.1 Systems and center of mass

A **system** is whatever you draw a boundary around: one block, two carts, a whole train. Forces between objects inside it are **internal**, and forces from anything outside are **external**. Only external forces change how the center of mass moves.



Force	System A	System B
Car's hitch on the trailer	internal	external
Road on the car's tires	external	doesn't act on B
Earth on the trailer	external	external

When the question doesn't care how the parts move relative to each other, treat the whole system as one object at its center of mass.

Exam trap

Saying the center of mass is closer to the "bigger" object. Only mass counts: a small lead ball and a big foam ball of equal mass have their center of mass halfway between them. In your answer, say "closer to the more massive object."

On the equation sheet

$$\bar{x}_{\text{cm}} = \frac{\sum m_i \bar{x}_i}{\sum m_i}$$

Each position counts in proportion to its mass. Work out x and y separately. On the exam you calculate it for at most five objects in a plane, or find it from symmetry.

Worked example

A 2.0 kg mass is at $x = 0$ and a 6.0 kg mass is at $x = 4.0$ m. Find the center of mass.

$$1 \quad x_{\text{cm}} = \frac{(2.0)(0) + (6.0)(4.0)}{2.0 + 6.0} = 3.0 \text{ m}$$

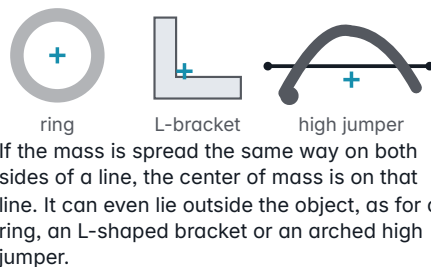
2.0 kg center of mass 6.0 kg

0 m 1 m 2 m 3 m 4 m

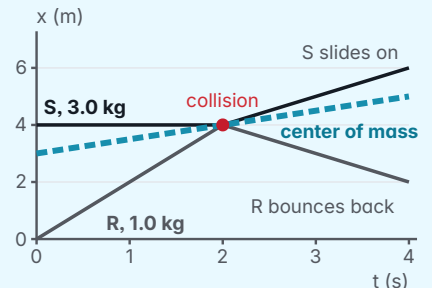
3.0 m 1.0 m

2 Check that the masses balance about it: $2.0 \times 3.0 = 6.0 \times 1.0$.

Where the center of mass is



A collision doesn't change how the center of mass moves



Glider R (1.0 kg) hits glider S (3.0 kg, at rest) at 2.0 m/s on a level, frictionless track, so no net external force acts. The center of mass moves from 3.0 m to 4.0 m in the first 2.0 s, and the internal collision forces don't change that 0.5 m/s.

Worked example

Find the center of mass of 1.0 kg at (0, 0), 1.0 kg at (4.0, 0) and 2.0 kg at (0, 3.0), in meters.

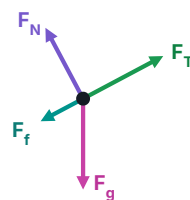
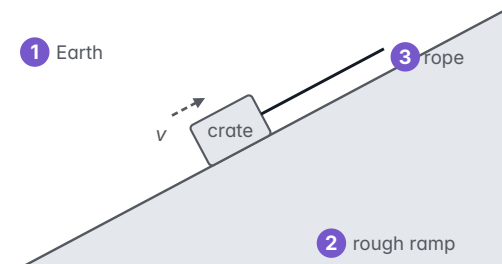
$$x \quad \frac{1.0(0) + 1.0(4.0) + 2.0(0)}{4.0} = 1.0 \text{ m}$$

$$y \quad \frac{1.0(0) + 1.0(0) + 2.0(3.0)}{4.0} = 1.5 \text{ m}$$

(1.0 m, 1.5 m). The 2.0 kg mass counts twice, so it pulls the point up toward itself.

2.2 Forces and what exerts them

Force	Exerted by	Direction	How big
F_g gravity	Earth or any mass	toward Earth's center	mg near Earth's surface
F_N normal	a surface	perpendicular to it, pushing	found from the other forces and the motion
F_T tension	a string or rope	along the string, pulling	the same all along an ideal string over ideal pulleys
F_f friction	a surface	along it, against slipping	up to $\mu_s F_N$ at rest; $\mu_k F_N$ sliding
F_s spring	a spring	back toward its relaxed length	k times the stretch



A crate pulled up a rough ramp at constant speed. Gravity from Earth (1) acts without touching. Then each thing touching the crate adds its forces: the ramp (2) pushes out of its surface (F_N) and along it (F_f), and the rope (3) pulls (F_T). Nothing else touches the crate, so nothing else goes on its diagram.

A force is a push or pull by one object on another, so you should be able to name the object behind every force you draw. An object can't push itself: a car speeds up because **the road** pushes its tires forward through static friction.

Exam trap

Drawing a force that nothing exerts.

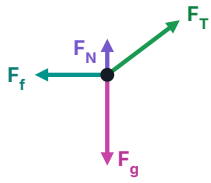
Every arrow needs an object doing the pushing or pulling. After a thrown ball leaves your hand, your hand exerts nothing on it. If you can't name what exerts a force, leave it off; that rules out "inertia" and a separate "centripetal force". A ball in flight gets one arrow: F_g , straight down.

Where contact forces come from

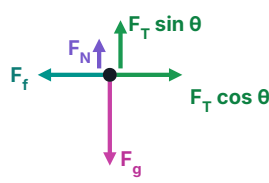
Normal, tension, friction and spring forces come from electric forces between atoms where things touch. Only gravity acts at a distance.

2.2 Drawing a free-body diagram

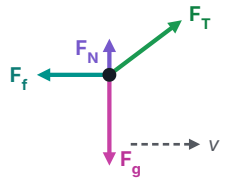
Five diagrams of one block pulled at constant speed by a rope at 37° above a rough floor. Only the first is right.



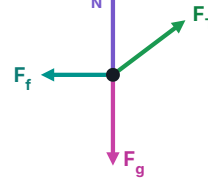
This one is right: four real forces, each drawn from the dot. The rope also pulls up a little, so F_N is shorter than F_g .



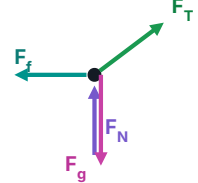
Components are drawn. Split F_T into parts in your equations, never on the diagram. Each force gets one arrow.



It has extra arrows. Velocity isn't a force, and nothing exerts a "force of motion", so neither belongs here.



The lengths are wrong. At constant velocity the up and down forces balance, $F_N + F_T \sin \theta = F_g$, so F_N must be shorter than F_g .



F_N doesn't start at the dot. Every arrow starts on the dot and points the way its force acts, even a push from below.

What a full-credit diagram needs

- 1 One **dot** for the object or system.
- 2 One **straight arrow for each force**, starting on the dot and pointing the way the force acts.
- 3 A **label** on every arrow: F_g , F_N , F_T , F_f , F_s , or a clear name.
- 4 Arrows that point the **same way** go side by side, not on top of each other, so both can be seen.
- 5 **No components**, velocity, acceleration or net force: graders read every arrow as a force.
- 6 If asked for **relative lengths**, draw them so the forces add up to the net force: with no vertical acceleration, up equals down.

Worked example

A rope pulls a 3.0 kg block across a rough floor with 10 N at 37° above the horizontal, and the block slides right. $\mu_k = 0.25$. Find its acceleration.

- 1 Vertical forces balance: $F_N = mg - F_T \sin 37^\circ = 30 - 10(0.60) = 24$ N
- 2 Friction: $F_f = \mu_k F_N = 0.25(24) = 6.0$ N
- 3 Horizontal: $F_T \cos 37^\circ - F_f = ma$, so $8.0 - 6.0 = 3.0a$

$$a = 0.67 \text{ m/s}^2 \text{ to the right}$$

Quick check

A crate rides in a truck that is braking, without sliding. Which forces act on the crate?

Gravity, the normal force, and static friction pointing backward. The crate slows down with the truck, so the net force on it points backward, and friction is the only force that can supply it.

Gravity on a ramp

On the diagram

In the equations



Take +x along the slope, the way the block accelerates. Gravity still points straight down on the diagram. Split it into $mg \sin \theta$ down the slope and $mg \cos \theta$ into the ramp only in your equations. The angle between F_g and the line perpendicular to the ramp is also θ , because tipping the ramp by θ tips that line by θ too.

Worked example

A 2.0 kg block slides down a frictionless 37° ramp. Find F_N and a .

- 1 Perpendicular: $F_N = mg \cos 37^\circ = 20(0.80) = 16$ N
- 2 Along the slope: $mg \sin 37^\circ = ma$, so $a = g \sin 37^\circ = 10(0.60)$

$$a = 6.0 \text{ m/s}^2 \text{ down the slope, for any mass}$$

Diagram and equations must match

Every force on the diagram must appear in your equations, and nothing else may. A force at an angle appears in both the x and y equations.

Exam trap

Writing $F_N = mg$ on a ramp. Only the part of gravity perpendicular to the ramp presses into it, so $F_N = mg \cos \theta$ when nothing else pushes into it. The 2.0 kg block above has $F_N = 16$ N, not 20 N.

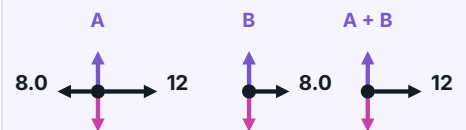
The normal force is not always mg

Situation	F_N
Level floor, nothing else vertical	mg
Pushed by a force F at θ below the horizontal	$mg + F \sin \theta$
Pulled by F at θ above the horizontal	$mg - F \sin \theta$
Ramp at angle θ	$mg \cos \theta$
Elevator speeding up going up, or slowing going down	$m(g + a)$
Top of a hill at speed v	$mg - \frac{mv^2}{r}$

Don't memorize this table. Each row comes from adding up the forces perpendicular to the surface, and writing that equation is what earns credit.

Choosing the system

12 N → [A] [B] A 1.0 kg, B 2.0 kg no friction



forces in newtons; vertical arrows: F_N up, F_g down
A 12 N push moves blocks A and B across a frictionless floor. Treat both as one system to get the acceleration: $a = 12/3.0 = 4.0 \text{ m/s}^2$. For the force between them, look at one block alone: A pushes B with $m_B a = 2.0(4.0) = 8.0$ N.

Quick check

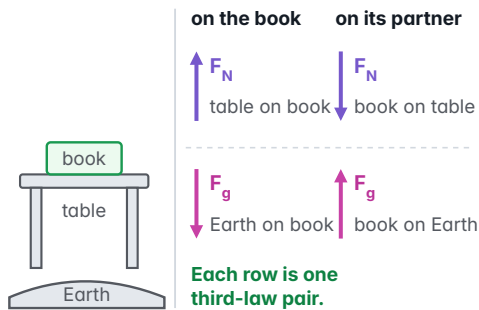
Now the 12 N push is on B, pushing left. Is the force between the blocks still 8.0 N?

No, 4.0 N. Now that force is the only sideways force on A, so it equals $m_A a = 1.0(4.0)$.

2.3 Newton's third law

When object A pushes or pulls on object B, B pushes or pulls back on A at the same moment. The two forces are equal in size, opposite in direction and of the same type, and they act on **different objects**.

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$



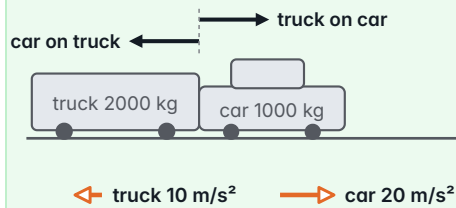
F_N and F_g on the book are equal here, but they are not a third-law pair, because both act on the book. In an accelerating elevator they stop being equal; the two forces of a real pair never do.

Quick check

A 50 kg skater and a 100 kg skater push off each other. Who pushes harder?

Neither. The forces are equal, so the 50 kg skater, with half the mass, gets twice the acceleration.

A truck and a car collide



Worked example

During the crash, the truck and the car each push on the other with 2.0×10^4 N. Find each acceleration.

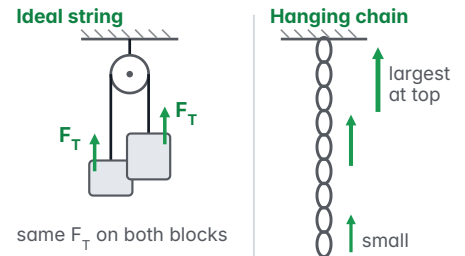
- Truck: $a = F/m = 2.0 \times 10^4 / 2000 = 10 \text{ m/s}^2$
- Car: $a = 2.0 \times 10^4 / 1000 = 20 \text{ m/s}^2$

A full-credit answer

Compare the forces the truck and the car exert on each other.

The force exerted on the car by the truck is equal in size and opposite in direction to the force exerted on the truck by the car, and both act for the same time. The car has less mass, so its velocity changes more.

Strings and pulleys



- An **ideal string** has no mass, so its tension is the same all along it, on both sides of an **ideal pulley** (no mass, no friction). It doesn't stretch, so objects tied to it move together.
- A **hanging chain** has the most tension at the top, which holds up the most chain. You'll explain this, not calculate it.

Exam trap

Thinking a third-law pair cancels out.

The two forces act on different objects, so each changes only its own object's motion. A cart pulls back on its horse as hard as the horse pulls it, yet the cart speeds up, because the horse's pull on the cart is bigger than the friction on the cart. To find how one object moves, add only the forces on that object.

2.4 Newton's first law

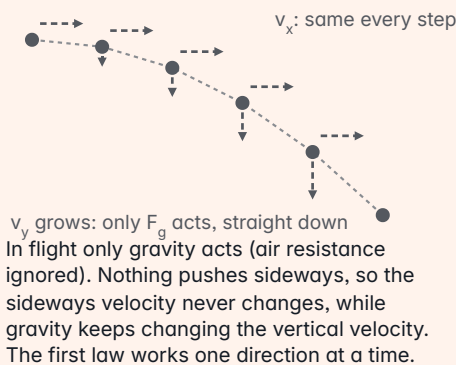
When the net force on an object is zero, its velocity stays constant: the same speed in the same direction. Staying at rest is the case where that velocity is zero. Either way the object is in **translational equilibrium**: its position changes at a steady rate, its velocity graph is flat, and its acceleration is zero.



Inertial frames

Newton's laws hold in an **inertial frame**, one that isn't accelerating. In a braking bus a cup slides forward with nothing pushing it: seen from the ground, the cup just keeps going while the bus slows under it. On the exam every frame is inertial unless the question says otherwise.

A thrown ball keeps its sideways speed



Worked example

A 2.0 kg sign hangs from two cables at 37° above the horizontal, one on each side. Find each tension.

- Sideways: the pulls cancel, so the tensions are equal.
- Vertical: $2F_T \sin 37^\circ = mg$, so $2F_T (0.60) = 20 \text{ N}$

$$F_T = 17 \text{ N in each cable}$$

Exam trap

Thinking motion needs a force. An object needs a net force only to speed up, slow down or turn. A sliding puck slows because friction pushes back on it, not because a "force of the push" runs out. Explain a motion by the force that changes it.

A full-credit answer

A pushed crate slides at constant velocity. Compare the push and the friction.

The crate's velocity is constant, so the net force on it is zero. That means the push and the friction are equal in size.

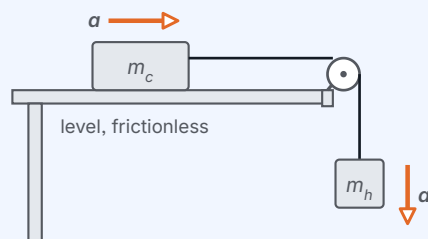
Quick check

A passenger who weighs 600 N stands on a scale in an elevator moving down at a steady 3 m/s. Does the scale read more than, less than, or exactly 600 N?

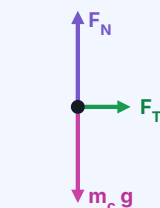
Exactly 600 N. The elevator isn't accelerating, so the scale pushes up with the passenger's full weight, whichever way it moves.

2.5 Newton's second law

⚡ A cart of mass m_c on a level, frictionless table, pulled by a hanging block of mass m_h



Cart



$$F_T = m_c a$$

Hanging block



$$m_h g - F_T = m_h a$$

System: cart + string + block, along the string



$$m_h g = (m_c + m_h) a$$

$$a = m_h g / (m_c + m_h)$$

(vertical forces on the cart cancel)

For the whole system, "along the string" bends around the pulley: only $m_h g$ pulls that way, and the tension forces inside the system cancel.

📄 On the equation sheet

$$\vec{a}_{\text{sys}} = \frac{\sum \vec{F}}{m_{\text{sys}}} = \frac{\vec{F}_{\text{net}}}{m_{\text{sys}}}$$

The center of mass of a system accelerates in the direction of the **net external force**.

Write one equation for each direction: $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$.

🔗 Solving any second-law problem

- 1 Pick the object or system the question is about.
- 2 Draw its free-body diagram.
- 3 Choose positive along each object's acceleration: right for the cart, down for the hanging block.
- 4 Write $\Sigma F = ma$ in that direction, one term per arrow, each with its sign.
- 5 Connect the objects (a taut string over a fixed pulley gives them equal accelerations), solve in symbols, check, then put in numbers.

📊 Worked example

The cart above is 2.0 kg and the hanging block is 1.0 kg. Find the acceleration and the tension.

- 1 Whole system: $a = \frac{m_h g}{m_c + m_h} = \frac{10}{3.0} = 3.3 \text{ m/s}^2$
- 2 Cart alone: $F_T = m_c a = 2.0(3.33) = 6.7 \text{ N}$
- 3 Check with the block: $m_h(g - a) = 1.0(10 - 3.33) = 6.7 \text{ N}$

$F_T = 6.7 \text{ N}$, less than the block's 10 N weight

🔍 Check that the answer makes sense

Try extreme cases: a tiny hanging mass ($m_h \rightarrow 0$) should give $a = 0$, and a massless cart should give free fall, $a = g$. A formula that fails this is wrong.

Accelerations worth recognizing

Setup	Acceleration
Two blocks over a pulley, $m_2 > m_1$	$\frac{(m_2 - m_1)g}{m_1 + m_2}$
Cart pulled by a hanging block	$\frac{m_h g}{m_c + m_h}$
The same, cart sliding toward the pulley with friction	$\frac{(m_h - \mu_k m_c)g}{m_c + m_h}$
Sliding down a rough ramp	$g(\sin \theta - \mu_k \cos \theta)$
Force F pushing blocks A and B on a frictionless floor	$\frac{F}{m_A + m_B}$

Use them to check answers. A derivation starts from Newton's second law.

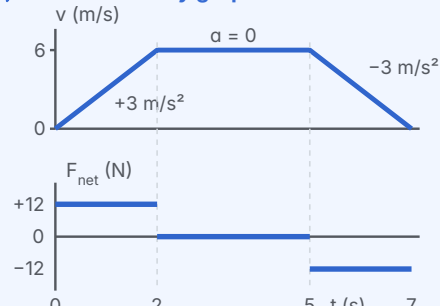
📊 Worked example

Blocks of $m_2 = 3.0 \text{ kg}$ and $m_1 = 2.0 \text{ kg}$ hang on either side of an ideal pulley. Find a and F_T .

- 1 $a = \frac{(3.0 - 2.0)(10)}{5.0} = 2.0 \text{ m/s}^2$
- 2 Lighter block, up positive: $F_T - m_1 g = m_1 a$, so $F_T = 2.0(10 + 2.0)$

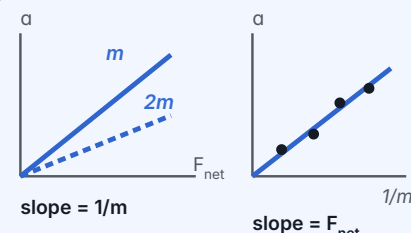
$F_T = 24 \text{ N}$, between the two weights (20 N and 30 N)

⚡ From a velocity graph to the net force



For a 4.0 kg cart, $F_{\text{net}} = ma$ with a the slope: 12 N forward, zero at constant speed, then 12 N back.

⚡ The second law as a graph



For one mass, a against F_{net} is a straight line through the origin with slope $1/m$, so twice the mass gives half the slope. Keep the force fixed and plot a against $1/m$: the slope is the force.

🔗 What happens if the hanging mass doubles?

Use your equation. Doubling m_h from 1.0 to 2.0 kg doubles the pull (the top of the fraction) but also adds mass to accelerate (the bottom), so a rises by less than double: from $10/3.0 = 3.3$ to $20/4.0 = 5.0 \text{ m/s}^2$.

⚠ Exam trap

- Saying the tension equals the hanging block's weight. That's only true at rest or at constant velocity. While the block speeds up downward, the net force on it points down, so the string pulls up with less than its weight: 6.7 N, not 10 N, in the example.
- Saying the net force points the way the object moves. It points the way the object accelerates. A cart rolling right while slowing down has a net force pointing left.

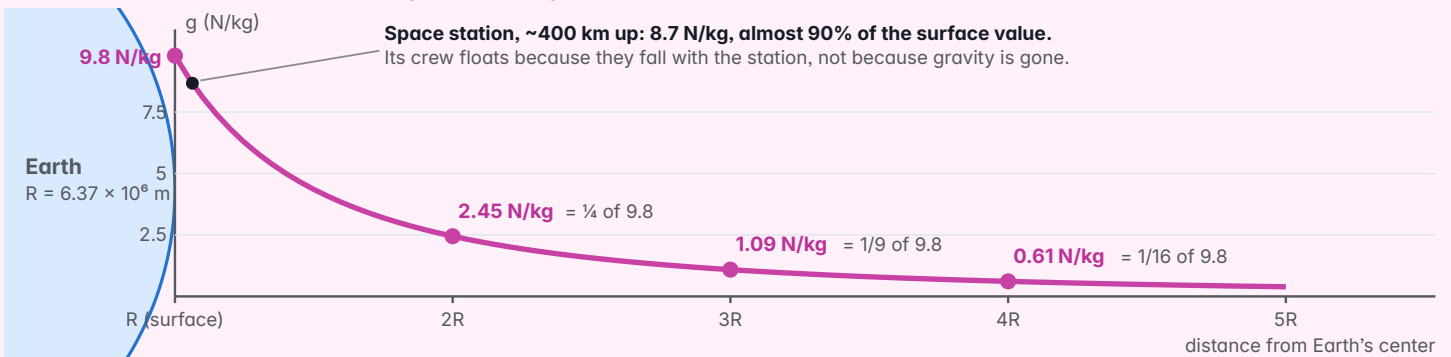
🔍 Quick check

A 1500 kg car speeds up uniformly from 0 to 20 m/s in 10 s on a level road. What net force acts on it?

3000 N forward: $a = 20/10 = 2.0 \text{ m/s}^2$, so $F = ma = 1500(2.0)$.

2.6 Gravitational force and apparent weight

⚡ Twice as far from Earth's center, a quarter of the pull



On the equation sheet

$$|\vec{F}_g| = G \frac{m_1 m_2}{r^2}$$

The equation sheet also gives $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$. The force always attracts, along the line between the centers, and each mass pulls the other equally hard.

Know by heart

Divide the force by the mass m of the object being pulled to get the field, $g = \frac{F_g}{m} = \frac{GM}{r^2}$. So an object's **weight** is $F_g = mg$, using the value of g where the object is (9.8 N/kg at Earth's surface).

Measure r from center to center: at height h above a planet of radius R , $r = R + h$. Near the surface g hardly changes, but thousands of kilometers up it does.

Worked example

Find g at Earth's surface, given $M = 5.97 \times 10^{24}$ kg and $R = 6.37 \times 10^6$ m.

$$1 \quad g = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})}{(6.37 \times 10^6)^2}$$

$g = 9.81 \text{ N/kg}$; the equation sheet gives 9.8

Exam trap

Using the height above the ground as r . A satellite 400 km up is $6.37 \times 10^6 + 4.0 \times 10^5 = 6.77 \times 10^6$ m from Earth's center. Putting in 4.0×10^5 m makes the force about 290 times too big.

Quick check

How far from Earth's center would your weight be half what it is at the surface?

$r = \sqrt{2}R \approx 1.41R$, about 2600 km up. Weight is proportional to $1/r^2$, so r^2 must double.

Ratio questions

Change	Result
Double the distance r	$F \times \frac{1}{4}$
Triple the distance	$F \times \frac{1}{9}$
Double both masses	$F \times 4$
Planet with $2M$, same R	$g \times 2$
Planet with the same M , $2R$	$g \times \frac{1}{4}$
Planet with $2M$ and $2R$	$g \times \frac{1}{2}$
Same density, $2R$ (volume and M grow 8 times)	$g \times 2$

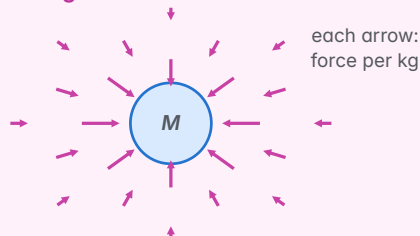
Write $g = GM/R^2$, multiply each quantity by its factor, and simplify. For $2M$ and $2R$: $\frac{2}{2^2} = \frac{1}{2}$.

Explaining a planet's gravity

A planet has Earth's mass but a larger radius. Does a dropped ball fall with a smaller acceleration there? "Gravity is weaker" earns nothing. This earns the point:

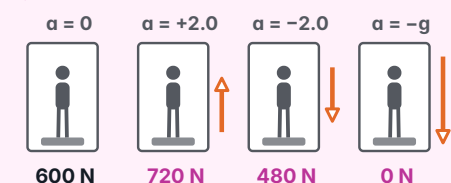
The gravitational field strength at the surface is smaller, because $g = GM/R^2$: M is the same and R is larger. Gravity is the only force on the ball, so its acceleration equals g and is smaller too.

⚡ The gravitational field



The field is the force on each kilogram at each point. It points toward the center and weakens as $1/r^2$. Two 1.0 kg masses 1.0 m apart attract each other with only 6.67×10^{-11} N, far less than their weights. That's why you draw only Earth's pull on everyday objects.

⚡ A scale reads the normal force



A 60 kg rider on a bathroom scale; each elevator's acceleration (m/s^2 , up positive) is above it and the reading below. A scale reads how hard it pushes up, F_N , not mg . With up positive (a is negative when it points down), $F_N - mg = ma$ gives

$$F_N = m(g + a)$$

Exam trap

Judging the scale reading by which way the elevator moves. Only the acceleration matters. A rider moving up but slowing down at 2.0 m/s^2 accelerates downward, so the scale reads less: $60(10 - 2.0) = 480 \text{ N}$. A rider moving down and speeding up also reads less.

Weightless doesn't mean no gravity

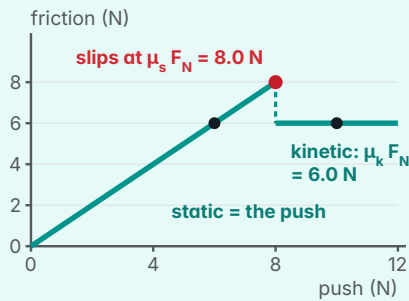
Your apparent weight is what a scale reads: the normal force on you. In free fall or orbit, gravity is the only force on you, so nothing pushes up, a scale would read zero, and you feel weightless while gravity still pulls. In a closed room, a scale can't tell standing on Earth from speeding up at 9.8 m/s^2 in space (the equivalence principle).

Two kinds of mass

Inertial mass resists acceleration; **gravitational mass** sets how hard gravity pulls. They are equal, which is why all objects fall with the same g .

2.7 Kinetic and static friction

⚡ Friction as you push harder



A 2.0 kg block on a level floor: $F_N = 20$ N, $\mu_s = 0.40$, $\mu_k = 0.30$. Static friction matches the push up to 8.0 N. Past that the block slides and friction drops to 6.0 N. μ_s is usually bigger than μ_k ; both depend on the two surfaces, not on contact area.

🔗 A lab question: measuring μ_k

Pull a block across a level floor at constant velocity with a force sensor, so the pull equals the friction. Add masses to the block to change $F_N (= mg \text{ here})$. Plot friction against F_N : the slope is μ_k .

📄 On the equation sheet

$$|\vec{F}_f| \leq |\mu \vec{F}_N|$$

It combines two rules. While an object **slides**, $F_f = \mu_k F_N$. While it **doesn't**, friction is whatever keeps it still, up to $\mu_s F_N$.

📊 Worked example

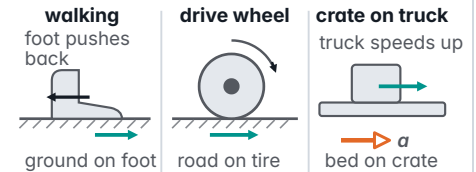
The block in the graph is pushed with 6.0 N, then 10 N. Find F_f and a .

- 6.0 N: static friction can give up to 8.0 N, so the block stays put. $F_f = 6.0$ N and $a = 0$.
- 10 N: more than 8.0 N, so it slides, and $F_f = \mu_k F_N = 6.0$ N.
- $a = \frac{10 - 6.0}{2.0} = 2.0 \text{ m/s}^2$

⚠ Exam trap

– Writing $F_f = \mu_s F_N$ for an object that isn't sliding. $\mu_s F_N$ is the most static friction can give, not what it gives. Pushed with 6.0 N, the block above feels 6.0 N of friction, not 8.0 N. Find static friction from the force balance.

Friction can point forward



Friction opposes the surfaces slipping past each other, not the motion. Your foot pushes back on the ground, so friction from the ground pushes you forward; without it your foot would slip backward.

📊 Worked example

A block on a 37° ramp is just about to slip. Find μ_s . Once it slides, with $\mu_k = 0.50$, find a .

- About to slip: $mg \sin \theta = \mu_s mg \cos \theta$, so $\mu_s = \tan 37^\circ = 0.75$
- Sliding: $a = g(\sin \theta - \mu_k \cos \theta) = 10(0.60 - 0.40)$

$a = 2.0 \text{ m/s}^2$; the mass cancels in both

2.8 Spring forces

📄 On the equation sheet

$$\vec{F}_s = -k\Delta\vec{x}$$

This is Hooke's law. Δx is the stretch or squeeze, measured from the **relaxed** length. The minus sign means the spring pulls or pushes back toward its relaxed length. k , in N/m, is the stiffness.

- An ideal spring has no mass and exerts equal, opposite forces on its ends.
- Graph the size of the force against the stretch and the slope is k . Graph the force with its sign against position and the slope is $-k$. Label your axes so it's clear which graph you drew.
- Real cords, like a bungee, may not follow Hooke's law.

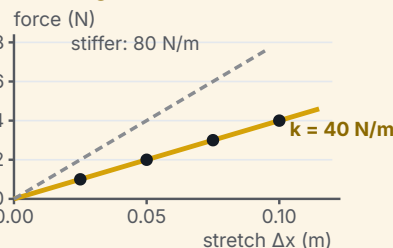
	String	Spring
Can it push?	No, only pull	Yes, when squeezed
Force set by	$\Sigma F = ma$	its stretch, $k \Delta x $

🔍 Quick check

A 0.50 kg mass hangs at rest from a spring with $k = 25$ N/m. How far is the spring stretched?

0.20 m: $k\Delta x = mg$, so $\Delta x = 5.0/25$.

⚡ Measuring k



Masses of 0.10 to 0.40 kg stretch the spring 2.5 to 10 cm. At rest $k\Delta x = mg$, so plot mg against Δx : the slope is $k = 40$ N/m.

🔗 Designing the experiment

Vary the hanging mass, measure each stretch from the relaxed length, repeat trials, and find k from the slope of the best-fit line, which averages out errors.

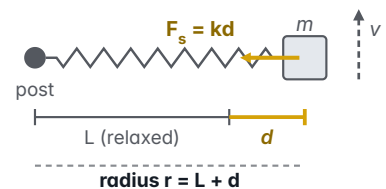
📊 Worked example

An elevator accelerates up at 2.0 m/s^2 . A 2.0 kg mass hangs from its spring scale ($k = 200$ N/m), moving with the elevator. Find the stretch.

- Up positive: $k\Delta x - mg = ma$
- $\Delta x = \frac{m(g+a)}{k} = \frac{2.0(10+2.0)}{200}$

$\Delta x = 0.12$ m, up from 0.10 m at rest

A spring force in a circle



A block circles a post on a frictionless table, held by a spring of relaxed length L stretched by d . The spring is the only inward force, so

$$kd = \frac{mv^2}{L + d}$$

📊 Worked example

With $m = 0.50$ kg, $k = 100$ N/m, $L = 0.20$ m and $d = 0.050$ m, find v .

- $F_s = kd = 5.0$ N; $r = L + d = 0.25$ m
- $v = \sqrt{F_s r / m} = \sqrt{5.0(0.25)/0.50}$

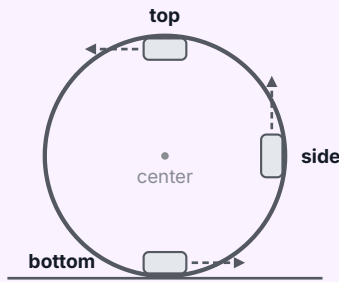
$v = 1.6 \text{ m/s}$

⚠ Exam trap

Using the spring's whole length in Hooke's law. The spring force depends only on the stretch: here $kd = 5.0$ N, not $k(L + d) = 25$ N. The radius is the whole length, $L + d$.

2.9 Circular motion

⚡ Free-body diagrams around a vertical loop, with toward the center as positive



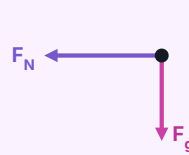
At the top



$$mg + F_N = mv^2/r$$

Both forces point to the center.
At the slowest speed, $F_N = 0$.

At the side



$$F_N = mv^2/r$$

F_N points to the center;
gravity slows the cart.

At the bottom



$$F_N - mg = mv^2/r$$

$F_N > mg$, so the rider
feels heavier.

⚠ Exam trap

Drawing the normal force upward at the top of a loop. At the top the track is above the cart and can only push away from itself, so it pushes down, toward the center. Draw F_g and F_N both down, side by side.

📐 Worked example

A 0.50 kg cart moves at 8.0 m/s at the top of a 2.0 m radius loop. Find F_N and the slowest speed that keeps it on the track.

- Inward force needed: $mv^2/r = 0.50(64)/2.0 = 16$ N
- Gravity supplies $mg = 5.0$ N of it, so $F_N = 16 - 5.0 = 11$ N, down
- Slowest: $F_N = 0$, so $mg = mv^2/r$ and $v = \sqrt{gr} = \sqrt{20}$

$F_N = 11$ N; slowest speed 4.5 m/s

📖 On the equation sheet

$$a_c = \frac{v^2}{r} \quad T = \frac{1}{f}$$

f is the frequency, in turns per second. Not on the sheet: $v = 2\pi r/T$, because in one period T the object goes once around, a distance $2\pi r$.

❓ Quick check

A wheel of radius 0.25 m turns at 2.0 revolutions per second. Find the speed of its rim and its centripetal acceleration.

3.1 m/s and 39 m/s²: $T = 0.50$ s,
 $v = 2\pi(0.25)/0.50$, $a_c = v^2/r$.

🔗 Why a rider feels heavier

At the bottom the net force points up, toward the center, so the upward normal force is bigger than the rider's weight, and the rider feels heavier.

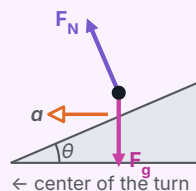
What provides the force toward the center

Situation	Inward force
Car on a flat curve	static friction
Satellite, the Moon	gravity
Ball on a string, level circle	part of F_T
Top of a loop	$F_g + F_N$
Over the top of a hill	$F_g - F_N$

Then set the total inward force equal to mv^2/r .

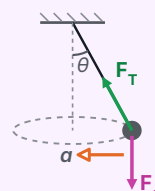
⚡ Banked curves and conical pendulums

Banked, no friction

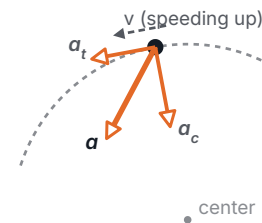


On a frictionless bank, $F_N \cos \theta = mg$ holds the car up and $F_N \sin \theta = mv^2/r$ turns it, so $\tan \theta = v^2/(rg)$. A conical pendulum works the same way, with F_T in place of F_N and θ from the vertical. Banks that need friction are only explained in words.

Conical pendulum



Speeding up on a circle

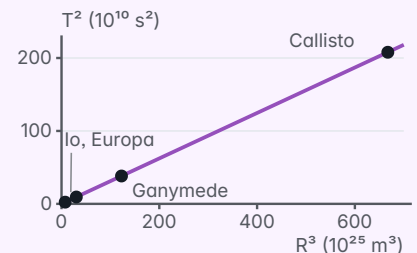


Changing direction takes a centripetal acceleration a_c toward the center; changing speed adds a tangential acceleration a_t along the path. The total acceleration is their vector sum. At constant speed $a_t = 0$, but the object still accelerates.

📖 Orbits: Kepler's third law

In a circular orbit of radius R , gravity is the only force toward the center:

$$\frac{GMm}{R^2} = \frac{m(2\pi R/T)^2}{R}, \text{ so } T^2 = \frac{4\pi^2}{GM} R^3$$



Jupiter's four big moons, whose orbits are nearly circular, fall nearly on one straight line on this graph. The slope is $4\pi^2/GM$, so $M = 4\pi^2/(G \times \text{slope}) = 1.90 \times 10^{27}$ kg. Kepler's first and second laws aren't on the exam.

⚠ Exam trap

Saying a "centrifugal force" throws you outward on a turn. No object exerts it. Your body tends to keep going straight, so in a left turn you slide right until the door pushes you inward, toward the center.

Unit 2 on the free-response questions

When the question says	What earns the point
"Draw and label the forces (not components) exerted on the block." (2.2)	Draw a dot, then one straight, labeled arrow for each real force, starting at the dot and pointing the way the force acts. Arrows that point the same way go side by side.
"Starting with Newton's second law, derive an expression for the acceleration in terms of m_1 , m_2 and g ." (2.5)	Write $\Sigma F = ma$ for a named object or system, put in each force with its sign, and solve. The final line may use only the symbols listed, so no F_T and no numbers.
"Is the tension greater than, less than, or equal to the weight? Justify." (2.5) (2.6)	Name the forces, say which way the acceleration (and so the net force) points, then compare: "It accelerates downward, so the net force points down, which means $F_g > F_T$." Scale readings work the same way.
"Compare the forces the two objects exert on each other." (2.3)	By the third law the two forces are equal in size and opposite in direction, and act for the same time. If no other large forces act, the object with less mass has the larger acceleration and change in velocity.
"Does the crate slide? Justify." (2.7)	Find the friction needed to keep the crate still, compare it with the most static friction can give, $\mu_s F_N$, and say whether it slides. Use $F_f = \mu_k F_N$ only once it is sliding.
"Explain why the cart stays on the track at the top of the loop." (2.9)	Name the real forces toward the center (F_g and F_N , both down) and set the net force toward the center equal to mv^2/r . The cart stays on while $F_N \geq 0$, which needs $v^2 \geq gr$.
"Is your equation consistent with your claim?" (2.5)	Point to the part of the equation that produces the change, for example: "Doubling m_h doubles the top of $a = m_h g / (m_c + m_h)$ but also makes the bottom bigger, so a less than doubles." If equation and claim disagree, say which was wrong.
"What would you graph to get a straight line, and how would you find μ_k (or k , or M)?" (2.7) (2.8) (2.9)	Rearrange the equation into $y = (\text{slope}) x$, name both axes with units, and say what the slope means: friction against F_N has slope μ_k , mg against Δx has slope k , and T^2 against R^3 has slope $4\pi^2/GM$, so $M = 4\pi^2/(G \times \text{slope})$.

Ⓢ The four free-response questions

- 1, Mathematical Routines (10 points): derive and calculate, often from a free-body diagram.
- 2, Translation Between Representations (12 points): draw a diagram, derive an equation and sketch a graph, then show they agree.
- 3, Experimental Design and Analysis (10 points): plan an experiment, then graph given data and use the slope.
- 4, Qualitative/Quantitative Translation (8 points): make a claim in words, derive an equation, and connect the two.

On the May 2027 exam, 42 multiple-choice questions take 85 minutes and these four take 95; each section is half the score.

🔗 Where Unit 2 comes back

- Unit 3 A force along the motion does work, $W = F_{\parallel} d$, and kinetic friction turns mechanical energy into thermal energy.
- Unit 4 The second law is rewritten with momentum. Because third-law pairs are equal and opposite, forces inside a system can't change its total momentum.
- Unit 5 Torque, the turning effect of a force, uses the same forces, drawn where they act on the object.
- Unit 7 The spring force drives oscillations, with period $T_s = 2\pi\sqrt{m/k}$.
- Unit 8 The buoyant force from a fluid is one more upward arrow on the diagram. An object's weight is the same in any fluid.

This unit's angles are in degrees. Test your calculator: $\sin 37^\circ$ should give 0.60. If it gives -0.64 , it is in radian mode and every ramp answer will be wrong.

📱 Set degree mode

TI-84 Plus CE. **mode**, arrow down to the row Radian Degree, highlight DEGREE, press **enter**, then **2nd mode** to quit.

TI-Nspire CX. **on** for the home screen, then 5 Settings, 2 Document Settings. Set Angle to Degree, move to Make Default and press **enter**.

Desmos. Open the wrench (graph settings) and choose Degrees.

📱 Enter powers of ten

TI-84 Plus CE. Type 6.67, then **2nd**, **(EE)**, then **(-)** 11. The screen shows 6.67E-11. Use **(-)** for a negative sign, never the subtraction key.

TI-Nspire CX. Type 6.67 **(x)** 10 **(^)** **(-)** 11. Use **(-)** for a negative sign, never the subtraction key.

Desmos. Type 6.67×10^{-11} .

⚠️ Small things that cost points

- For "calculate", write the equation with your numbers in it, then the answer with units.
- For a slope, put two points from your best-fit line into the slope formula, then give the value with units.
- A derivation starts from a law like $\Sigma F = ma$ and ends with only the allowed symbols.

🔗 A model answer for question 4: claim, derive, connect

The question. Blocks of 3.0 kg and 2.0 kg hang over an ideal pulley. A student claims that adding 1.0 kg to each block leaves the acceleration unchanged, because the difference in weight stays the same. Is the student right?

Claim, in words. No: the acceleration gets smaller. The unbalanced force, the difference in the weights, is unchanged, but it now has to accelerate more total mass.

Derive. Newton's second law for the system along the string: $(m_2 - m_1)g = (m_1 + m_2)a$, so $a = \frac{(m_2 - m_1)g}{m_1 + m_2}$.

Connect. The numerator stays $(1.0 \text{ kg})g$ while the denominator grows from 5.0 to 7.0 kg, so a drops from 2.0 to 1.4 m/s². This matches the claim: the acceleration gets smaller.