

Unit 3

AP Physics 1

Work, Energy, and Power

Kinetic energy · Work · Potential energy · Conservation of energy · Power

Unit 1
10–15%
Kinematics

Unit 2
18–23%
Force and
dynamics

Unit 3
18–23%
Work, energy,
power

Unit 4
10–15%
Linear
momentum

Unit 5
10–15%
Torque and
rotation

Unit 6 5–8%
Rotating
systems

Unit 7 5–8%
Oscillations

Unit 8
10–15%
Fluids

Unit 3 is 18–23% of the multiple-choice section. Any of the four free-response questions can test energy, through bar charts, graphs of energy against position, derivations or a lab.

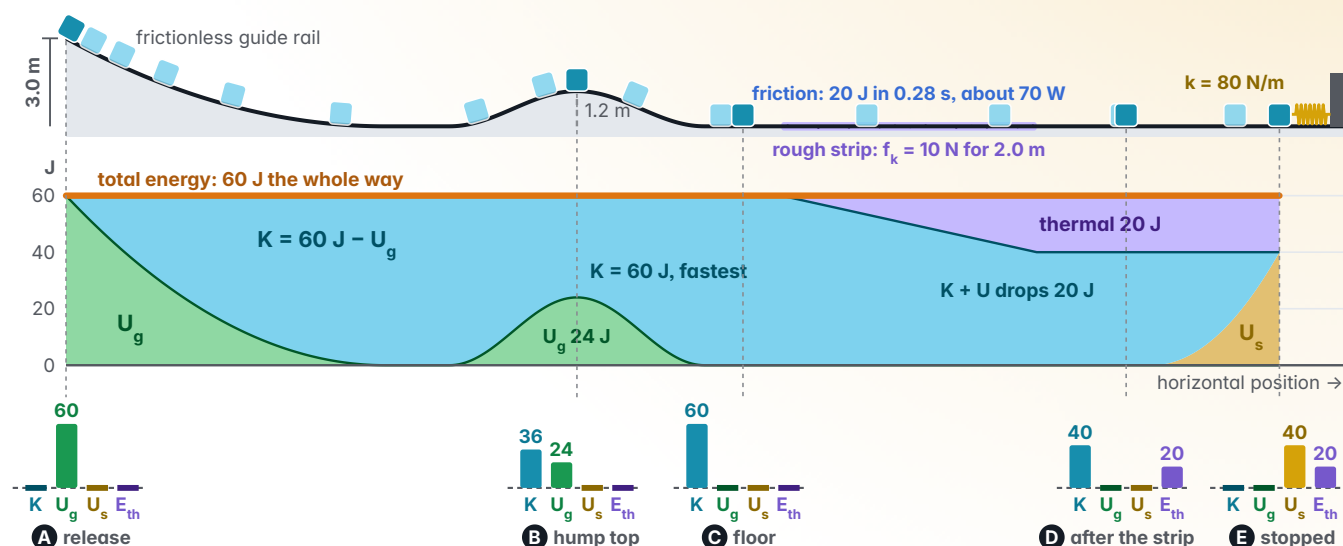
3.1
Translational kinetic
energy

3.2
Work

3.3
Potential energy

3.4
Conservation of energy

3.5
Power



A 2.0 kg block on a guide rail, frictionless except for the rough strip, starts from rest at A and stops for an instant against the spring at E ($g = 10 \text{ m/s}^2$). Pale blocks show it every 0.15 s. The system is the block, Earth, spring and strip; E_{th} is thermal energy.

What the exam asks most

- Energy bar charts for a named system.** Draw a U_g bar only if Earth is in the system, and a U_s bar only if the spring is. With no outside work and no friction, the bars always add to the same total. Draw a zero energy as a short line on the zero line, so the grader sees it. (3.4)
- A derivation from conservation of energy.** Write $K_i + U_i = K_f + U_f$, cross out the zero terms and say why ("starts from rest"), then put in the problem's quantities, such as the drop $d \sin \theta$ for a block sliding d down a ramp. (3.4)
- Energy against position.** With no friction, the mechanical energy E is a flat line. Down a straight ramp U_g falls in a straight line, U_s is a parabola starting where the block meets the spring, and $K = E - U_g - U_s$ is what's left. (3.4)

- The work one force does, with its sign.** Only the part of the force along the motion counts: the work is positive if that part points with the motion, negative if against it, and zero if the force is at right angles. For a changing force, the work is the area under its graph against position. (3.2)
- Friction and thermal energy.** Kinetic friction turns $f_k d$ of mechanical energy into thermal energy, with d the length of the path slid. A block that slides 3.0 m against 4.0 N of friction loses 12 J. (3.2) (3.4)
- Doubling the speed, or halving the time.** Twice the speed gives four times the kinetic energy, so the same brakes need four times the distance. The same lift in half the time takes twice the power. (3.1) (3.5)

On the equation sheet

$$K = \frac{1}{2}mv^2 \quad W = F_{\parallel}d = Fd \cos \theta$$

$$\Delta K = \sum W_i = \sum F_{\parallel,i} d_i$$

$$U_s = \frac{1}{2}k(\Delta x)^2 \quad \Delta U_g = mg\Delta y$$

$$U_g = -\frac{Gm_1m_2}{r} \quad P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = F_{\parallel}v = Fv \cos \theta$$

U_g (capital G) is for gravity at any distance. The sheet also lists $g = 9.8 \text{ m/s}^2$, G , and $\sin$ and $\cos$ of 37° and 53° ; you may use $g = 10 \text{ m/s}^2$, as every example here does.

Not on the equation sheet

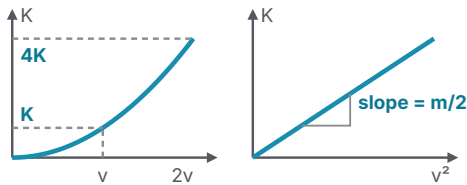
- Mechanical energy is $E = K + U$. With no outside work and no friction, $K_i + U_i = K_f + U_f$.
- With friction, $K_i + U_i = K_f + U_f + f_k d$, where d is the length of the path slid.
- Work by gravity or a spring: $W = -\Delta U$.
- A block sliding d down a ramp at angle θ drops a height $d \sin \theta$.
- $1 \text{ J} = 1 \text{ N}\cdot\text{m} = 1 \text{ kg}\cdot\text{m}^2/\text{s}^2$; $1 \text{ W} = 1 \text{ J/s}$.

3.1 Translational kinetic energy

Kinetic energy is the energy an object has because it moves. It uses the speed squared, so it has no direction and is never negative. Twice the mass gives twice the energy, but three times the speed gives nine times as much.

On the equation sheet

$$K = \frac{1}{2}mv^2$$



Worked example

A 1,500 kg car speeds up from 10 m/s to 20 m/s. How much kinetic energy does it gain?

- At 10 m/s: $\frac{1}{2}(1500)(10)^2 = 75,000 \text{ J}$
- At 20 m/s: $\frac{1}{2}(1500)(20)^2 = 300,000 \text{ J}$

$\Delta K = 225 \text{ kJ}$, three times the 75 kJ it had at 10 m/s

Kinetic energy has no direction and depends on the observer

- Two 3.0 kg carts roll toward each other at 4.0 m/s. Each has 24 J, so together they have **48 J**, though their momenta cancel.
- Its value depends on who measures the speed. A 0.50 kg ball thrown forward at 10 m/s on a train going 20 m/s has 25 J for a passenger, but **225 J** for someone on the platform, who sees it at 30 m/s. Both are right.

Know by heart

Kinetic energy from momentum.

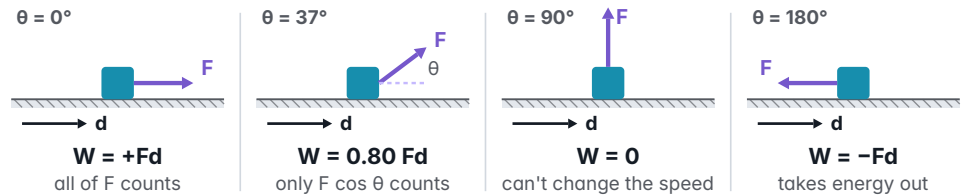
Because $p = mv$, $K = p^2/2m$. At the same momentum the lighter object has more kinetic energy: 4.0 kg·m/s is 16 J for a 0.50 kg ball, 4.0 J for a 2.0 kg cart.

Exam trap

Thinking twice the speed means twice the energy. K depends on v^2 , so square the speed ratio: twice the speed gives $2^2 = 4$ times the energy. A car at 8 m/s needs four times the braking distance it needs at 4 m/s, not twice.

3.2 Work

Work is energy that a force adds to an object or takes away from it as the object moves. Only the part of the force along the motion counts.



On the equation sheet

$$W = F_{\parallel}d = Fd \cos \theta$$

θ is the angle between the force and the direction the object moves. It is not the ramp angle, unless the two happen to match.

A block slides a distance d down a rough ramp at angle θ

Gravity	$+mgd \sin \theta$, from the part of gravity along the ramp, $mg \sin \theta$
Normal force	0, because it is perpendicular to the motion
Friction	$-f_k d$, because it points against the motion
Total	$\Delta K = mgd \sin \theta - f_k d$

Three ways a force does zero work

- Nothing moves.** Holding a barbell still tires your arms, but no energy goes into the barbell.
- The force is at right angles to the motion.** The floor's push on a rolling cart and the string on a ball whirled in a circle can't change the speed.
- The spot where the force acts doesn't move.** When you push off a wall, the wall pushes on your hand, which stays put. So the wall does no work on you: your speed comes from your muscles.

Exam trap

Giving work a direction. Work is an amount of energy, so it has a sign but no direction. Write "the rope does +1,200 J of work", not "1,200 J to the right". Plus means the force added energy; minus means it took energy away.

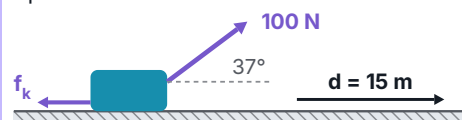
Quick check

You carry a 10 kg box 5.0 m across a room, then lift it 1.2 m onto a shelf, both at a steady speed. How much work does your hand do in each part?

Answer: carrying, **0 J**, because your hand pushes up and the box moves sideways. Lifting, $mg\Delta y = (10)(10)(1.2) = 120 \text{ J}$.

Worked example

A rope pulls a 20 kg sled from rest 15 m across level snow with 100 N at 37° above the horizontal; $\mu_k = 0.20$. Find the work done by each force and the final speed.



- Rope: $W = Fd \cos \theta = (100)(15)(0.80) = +1,200 \text{ J}$
- The rope also pulls up with $100 \sin 37^\circ = 60 \text{ N}$, so the snow pushes up with only $200 - 60 = 140 \text{ N}$. Friction is $0.20(140) = 28 \text{ N}$, and its work is $-28(15) = -420 \text{ J}$
- Gravity and the normal force do no work, so $\Delta K = 1,200 - 420 = 780 \text{ J} = \frac{1}{2}(20)v^2$

$$v = \sqrt{78} \approx 8.8 \text{ m/s}$$

Exam trap

Doing trig in radian mode. In radian mode, $\cos 37$ means the cosine of 37 radians, 0.77, not $\cos 37^\circ = 0.80$, and the rope's work comes out 1,150 J. Set degree mode before you start.

Degree mode on your calculator

TI-84 Plus CE. `mode`, arrow down to the row Radian DEGREE, highlight DEGREE, press `enter`, then `2nd mode` to quit.

TI-Nspire CX. `on` for the home screen, then 5 Settings, 2 Document Settings. Set Angle to Degree, move to Make Default and press `enter`.

Desmos. Open the wrench (graph settings) and choose Degrees.

At constant speed the total work is zero

You pull a 50 kg crate 4.0 m up a frictionless 30° ramp at a steady speed. Gravity does $-(50)(10)(4.0)(0.50) = -1,000 \text{ J}$. The speed doesn't change, so $\Delta K = 0$ and your pull must do **+1,000 J**.

3.2 Work, continued

The work-energy theorem

On the equation sheet

$$\Delta K = \sum W_i = \sum F_{\parallel,i} d_i$$

For an object you can treat as a point, add the work of every force, with signs: the total is the change in kinetic energy. A positive total means it speeds up; zero means its speed doesn't change.

Worked example

Pucks of 1.0 kg and 4.0 kg rest on frictionless ice. Each is pushed with 10 N for 2.0 m. Compare their kinetic energies and speeds.

- 1 Same force, same distance: $W = (10)(2.0) = 20 \text{ J}$ on each
- 2 Each has $K = 20 \text{ J}$, so $v = \sqrt{2K/m}$ is 6.3 m/s or 3.2 m/s

Same K; the heavier puck moves at half the speed

Same distance or same time

The same force over the same **distance** does the same work, so it gives the same change in kinetic energy. For the same **time**, it gives the same impulse and the same change in momentum (Topic 4.2). Check which one the question keeps fixed.

Worked example

A 12,000 kg truck with failed brakes enters a level gravel lane at 25 m/s. The gravel pushes back with a quarter of the truck's weight. How long must the lane be?

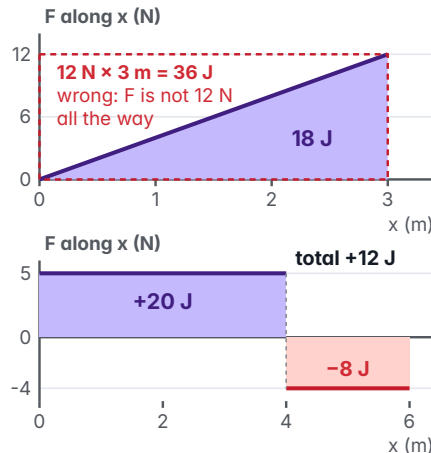
- 1 Only the gravel's push does work, against the motion:
 $-\frac{1}{4}mgd = 0 - \frac{1}{2}mv^2$
- 2 The mass cancels: $d = \frac{2v^2}{g} = \frac{2(25)^2}{10}$

125 m for a truck of any mass. At 50 m/s it needs 500 m, four times as long

Exam trap

Counting gravity twice. Gravity can go into your equation as work (if Earth is outside your system) or as a change in U_g (if Earth is inside), never both. A 2.0 kg ball falls 5.0 m: either gravity does +100 J of work or U_g drops 100 J, and the ball gains 100 J of K . Using both gives 200 J.

Work is the area under a force graph



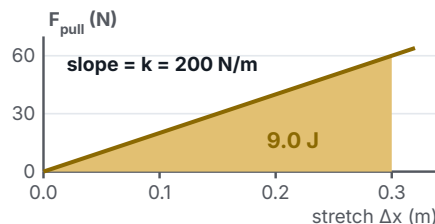
Area above the axis is positive work; area below it is negative.

Worked example

The force in the top graph pushes a 2.0 kg cart from rest. No other force does work. What is its final speed?

- 1 Area: $W = \frac{1}{2}(3.0)(12) = 18 \text{ J}$
- 2 $18 = \frac{1}{2}(2.0)v^2$, so $v = \sqrt{18}$

$v \approx 4.2 \text{ m/s}$. The final 12 N times the whole 3.0 m gives 36 J, twice the real work



Stretch a spring slowly and your pull grows from 0 to $k\Delta x$. Its work is the triangle, $\frac{1}{2}(\Delta x)(k\Delta x) = \frac{1}{2}k(\Delta x)^2$, and the spring stores it as U_s .

When the force changes

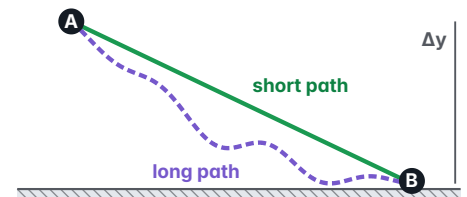
Use the area, not the final force times the distance. A compound bow's pull rises and then falls as you draw it back, so the energy it stores is the whole area under its force curve, not the force at full draw times the draw length.

Quick check

A spring ($k = 200 \text{ N/m}$) squeezed 0.30 m launches a 0.50 kg cart along a frictionless track. How fast does the cart leave?

Answer: the stored energy is the triangle above, 9.0 J. $\frac{1}{2}(0.50)v^2 = 9.0$, so **6.0 m/s**.

Work that depends only on the start and end



From A to B, gravity does $+mg|\Delta y|$ on both paths. If friction is the same size on both, the longer path turns more energy into thermal energy.

	Conservative	Nonconservative
Examples	gravity, spring force	friction, air resistance
Work	depends only on the start and end; zero around a closed loop	depends on the path
Potential energy	has one, and $W = -\Delta U$	has none; turns K or U into thermal energy and sound

Know by heart

Thermal energy from sliding. When an object slides against a constant friction force f_k over a surface that stays still, $\Delta E_{\text{th}} = f_k d$. Here d is the length of the path slid, not the straight-line distance from start to end.

Worked example

A 60 kg skier starts from rest and drops 50 m by one of two routes, 300 m or 500 m long, against 20 N of friction. Find K at the bottom.

- 1 Gravity, either route: $mg|\Delta y| = (60)(10)(50) = 30,000 \text{ J}$
- 2 Friction: $-20(300) = -6,000 \text{ J}$, or $-20(500) = -10,000 \text{ J}$

$K = 24 \text{ kJ}$ after the short route, 20 kJ after the long one

Exam trap

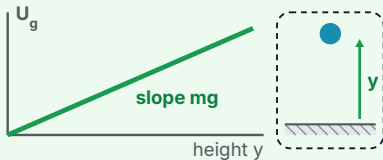
Thinking a ramp saves work. Raising a 50 kg box 1.0 m at a steady speed takes $mg\Delta y = 500 \text{ J}$ either way (no friction). Lifting straight up is 500 N over 1.0 m; a 2.0 m ramp is 250 N over 2.0 m. The ramp needs less force over a longer distance, so use $mg\Delta y$ whatever the path.

3.3 Potential energy

Potential energy is stored in the arrangement of a system whose parts pull or push on each other, by gravity or through a spring. A single object has none. All three formulas below are on the equation sheet; U_g (small g) is for near a surface, U_G (capital G) for any distance.

Gravity near a surface

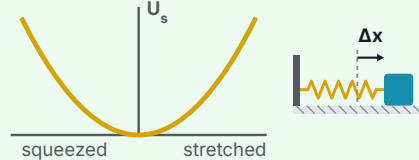
$$\Delta U_g = mg\Delta y$$



It belongs to the object–Earth system. Put the zero at any height you like: only changes in U_g matter.

A spring

$$U_s = \frac{1}{2}k(\Delta x)^2$$



It is stored in the spring. It is zero at the relaxed length and never negative, because Δx is squared.

Gravity at any distance

$$U_G = -\frac{Gm_1m_2}{r}$$



It applies to two spheres, with r measured center to center. It is zero when they are infinitely far apart, so it is negative at every real distance.

Potential energy belongs to a system

Say "the ball–Earth system's U_g ", not "the ball's U_g ". Leave Earth out of the system and there is no U_g ; gravity does work on the ball from outside instead.

Three spring equations not to mix up

Force	$F_s = -k\Delta x$	2.8
Energy	$U_s = \frac{1}{2}k(\Delta x)^2$	3.3
Period	$T_s = 2\pi\sqrt{m/k}$	7.2

Worked example

Squeeze a 400 N/m spring by 0.10 m, then by 0.20 m. Find U_s each time.

- $\frac{1}{2}(400)(0.10)^2 = 2.0 \text{ J}$
- $\frac{1}{2}(400)(0.20)^2 = 8.0 \text{ J}$

Doubling the squeeze quadruples the energy

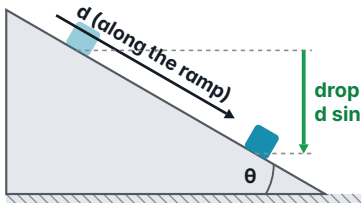
Worked example

How much does U_G rise when a 1,000 kg satellite goes from Earth's surface ($R = 6.4 \times 10^6 \text{ m}$) to $2R$ from Earth's center? $M = 6.0 \times 10^{24} \text{ kg}$.

$$\begin{aligned} 1 \quad \Delta U_G &= -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R} \\ 2 \quad &= \frac{(6.67 \times 10^{-11})(6.0 \times 10^{24})(1000)}{2(6.4 \times 10^6)} \end{aligned}$$

$\approx 3.1 \times 10^{10} \text{ J}$. Using $mg\Delta y$ with $\Delta y = R$ would give twice this, because g weakens with height

The height a block drops on a ramp



Worked example

A 1.5 kg block slides 4.0 m down a 30° ramp. Find ΔU_g and the work gravity does.

- $\Delta y = -(4.0) \sin 30^\circ = -2.0 \text{ m}$
- $\Delta U_g = (1.5)(10)(-2.0) = -30 \text{ J}$

U_g falls 30 J, and gravity does +30 J of work. That is $W = -\Delta U$, true for gravity and springs

You choose where U_g is zero

Lift a 0.50 kg book from the floor to a 2.0 m shelf. With the zero at the floor, U_g goes from 0 to +10 J; with the zero at the shelf, from -10 J to 0. Either way it rises 10 J.

Exam trap

Using the length of the ramp as the height. A block that slides 3.0 m down a 30° ramp drops only $3.0 \sin 30^\circ = 1.5 \text{ m}$, so putting 3.0 m into $mg\Delta y$ doubles the energy. Use the vertical drop, $d \sin \theta$.

Exam trap

Measuring Δx from the wrong place.

Δx is how far the spring is stretched or squeezed from its relaxed length, not the spring's length. A 0.30 m spring stretched to 0.36 m has $\Delta x = 0.06 \text{ m}$.

Squaring only part of Δx . A squeeze of $2x$ stores $\frac{1}{2}k(2x)^2 = 2kx^2$. Writing $\frac{1}{2}k \cdot 2x^2 = kx^2$ loses a factor of 2. Put the whole Δx in brackets before you square it.

Graphs that come out as straight lines

U_g against height y	slope mg
U_s against $(\Delta x)^2$	slope $k/2$
K against v^2	slope $m/2$

For U_s and K , plot against the squared quantity to get a straight line; the slope gives the constant.

Powers of ten on your calculator

TI-84 Plus CE. Type 6.67, then $\boxed{2\text{nd}} \boxed{,}$ (EE), then $\boxed{(-)} \boxed{11}$. The screen shows 6.67E-11. Use $\boxed{(-)}$ for a negative sign, never the subtraction key.

TI-Nspire CX. Type 6.67 $\boxed{\times} \boxed{10} \boxed{\wedge} \boxed{(-)} \boxed{11}$. Use $\boxed{(-)}$ for a negative sign, never the subtraction key.

Desmos. Type $6.67 \cdot 10^{-11}$.

Know by heart

Three or more objects. Add the potential energy of every pair, once each. Three stars give three terms like $-Gm_1m_2/r_{12}$, one for each pair.

Exam trap

Comparing negative energies by their size. $-5 \times 10^9 \text{ J}$ is less than $-2 \times 10^9 \text{ J}$, just as -5 is less than -2 . So a satellite moving out from the first to the second gains $3 \times 10^9 \text{ J}$ of U_G . Compare signed values, as on a number line.

3.4 Conservation of energy

Energy is never created or destroyed. In this unit, a system's total energy stays constant when the outside forces on it do zero net work. So always say first which objects are in the system. The system you choose tells you which energies to count.

System and its energies	Total stays constant when
Block (K)	zero net outside work
Block + Earth (K) (U _g)	no friction and zero net outside work
Block, Earth and spring (K) (U _g) (U _s)	no friction and zero net outside work
Block, Earth, spring and rough floor (K) (U _g) (U _s) (E _{th})	zero net outside work

Know by heart

When mechanical energy stays constant. Both must be true:

- 1 Outside forces do zero net work on the system.
- 2 Nothing inside turns $K + U$ into other forms, such as thermal energy from friction or drag.

Then $K_i + U_i = K_f + U_f$. With friction inside the system, $K_i + U_i = K_f + U_f + f_k d$.

Conserved versus constant

Energy is always **conserved**: it is never made or destroyed. **Constant** is a claim about the system you picked, true when no net energy crosses its boundary. With the box alone as the system, a falling box gains K , so its energy isn't constant, yet energy is conserved: Earth's gravity did work on it.

Model answer: constant energy

"No outside force does work on the cart-spring system, because the wall and the floor don't move, and the track is frictionless, so $K + U_s$ stays constant." Name your own system and say why no outside force does work.

Energy in a pole vault

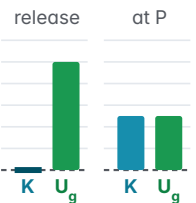
The bent pole stores the run-up's kinetic energy as U_s , then turns it into U_g . If all the K of a 10 m/s run-up became U_g , the center of mass would rise $v^2/2g = 5.0$ m, whatever the mass.

Drawing energy bar charts

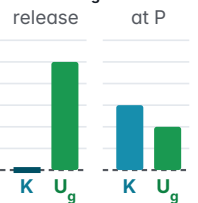
What earns the bar-chart points

- Start each bar at the zero line and make its height match the scale.
- Draw a zero energy as a short line on the zero line, not a blank, so the grader knows you meant zero.
- If the system's energy is constant, the bars add to the same total at every moment. A bar below the line counts as negative.
- Draw only the energies the named system has: no U_g bar if Earth is outside it.

Wrong: split evenly



Right: U_g = 2, K = 3



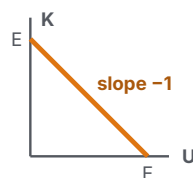
The block is released from rest 5 units up a frictionless track and passes P, 2 units up. There $U_g = 2$, so $K = 3$: an even split is wrong.

Exam trap

- Giving kinetic energy to something that isn't moving. At release and at maximum compression, $K = 0$: cross the term out and say why. A block dropped from rest a height h onto a spring that it squeezes by x gives $mg(h + x) = \frac{1}{2}kx^2$, with no K on either side.
- Saying energy is "lost" on a frictionless track. Nothing turns it into thermal energy, so the total stays the same. Say what it turns into: on the way up a loop, K turns into U_g , and back again on the way down.

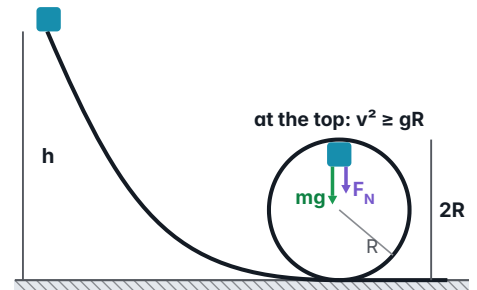
A pendulum can't swing back higher

Let a heavy pendulum go from rest, touching your nose. The bob-Earth system can't gain energy without an outside push, so the bob can't swing back past your nose; air drag makes it stop just short.



When E is constant, a graph of K against U is a straight line from $(0, E)$ to $(E, 0)$. Write " $K + U$ stays constant", not " K and U are the same": they are equal at only one point.

Deriving the release height for a loop



A block slides from rest down a frictionless track into a loop of radius R . What is the lowest release height h that keeps it on the track at the top?

Worked example

Derive the lowest h in terms of R .

- 1 Block-Earth, no friction: $K_i + U_i = K_f + U_f$
- 2 $0 + mgh = \frac{1}{2}mv^2 + mg(2R)$
- 3 At the top, gravity and F_N both point down, toward the center: $mg + F_N = mv^2/R$. Just staying on means $F_N = 0$, so $v^2 = gR$
- 4 $mgh = \frac{1}{2}m(gR) + 2mgR$

$h = 2.5R$ (for $R = 0.40$ m, $h = 1.0$ m). Released from $2R$, it could at best reach the top with zero speed, less than the $\sqrt{gR}$ it needs, so it leaves the track before the top

How to earn the derivation points

- 1 Start from a principle written in symbols, such as $K_i + U_i = K_f + U_f$ or $\Delta K = \sum W$.
- 2 Cross out each term that is zero and say why ("starts from rest").
- 3 Put in the problem's own quantities. A block of mass M that starts from rest, slides a total distance L down a frictionless ramp and stops after squeezing a spring by x gives $MgL \sin \theta = \frac{1}{2}kx^2$.
- 4 Solve using only the symbols the question allows.

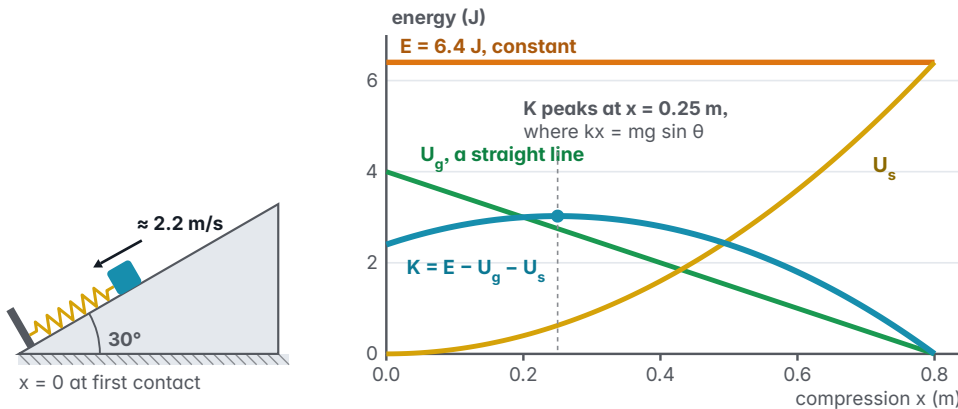
Exam trap

Measuring heights from different zeros.

A block starts $6R$ above the ground and enters a loop whose bottom is $2R$ above the ground. Measured from the ground, the loop's top is at $4R$, so the block has dropped $6R - 4R = 2R$. Taking the top as $2R$ (measured from the loop's bottom) wrongly gives a drop of $4R$. Measure every height from one zero.

3.4 Conservation of energy, continued

Graphs of energy against position



A 1.0 kg block slides down a frictionless 30° ramp and meets a spring ($k = 20 \text{ N/m}$) at about 2.2 m/s. For the first 0.25 m of squeeze it keeps **speeding up**, because it loses U_g faster than the spring gains U_s .

Model answer: speeding up after it meets the spring

"From $x = 0$ to 0.10 m the block-Earth-spring system loses 0.50 J of U_g and gains 0.10 J of U_s . Its mechanical energy is constant, so K rises by 0.40 J and the block speeds up."

Here $\Delta U_g = -mgx \sin \theta = -(1.0)(10)(0.10)(0.50)$ and $U_s = \frac{1}{2}(20)(0.10)^2$. An answer that mentions only one of the two potential energies is incomplete.

Exam trap

- Assuming the block slows as soon as it touches the spring. At first the spring pushes back only weakly, so compare the two potential-energy changes before you say whether it speeds up. Here the block speeds up until $x = 0.25 \text{ m}$.
- Drawing U_g against position as a curve. On a straight ramp U_g drops the same amount for every meter, so it is a straight line. Against time it can curve, because the block speeds up.
- Drawing the total energy sloping down. With no outside work and no friction, nothing adds or removes energy, so the total is a horizontal line: in the graph above it stays at 6.4 J while K , U_g and U_s change.

Quick check

Use the graph: what are K and the speed at $x = 0.50 \text{ m}$? Where is the block when $U_s = U_g$?

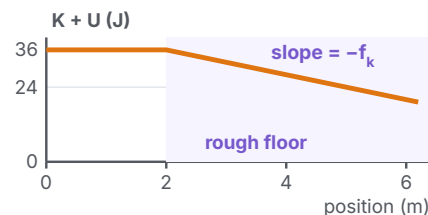
Answer: $K = 6.4 - 1.5 - 2.5 = 2.4 \text{ J}$, so 2.2 m/s , its speed at first contact. They are equal at $x \approx 0.43 \text{ m}$, where the curve $U_s = 10x^2$ meets the line $U_g = 5(0.8 - x)$.

Worked example

A 2.0 kg block slides from rest down a frictionless 1.8 m drop, then 3.0 m across a floor with $\mu_k = 0.20$. Find its speed after the 3.0 m of floor, and how far it would slide on the floor in all.

- $mgh = (2.0)(10)(1.8) = 36 \text{ J}$
- Thermal energy: $\mu_k mgd = 0.20(2.0)(10)(3.0) = 12 \text{ J}$
- $K_f = 36 - 12 = 24 \text{ J} = \frac{1}{2}(2.0)v^2$
- To stop: $mgh = \mu_k mgd$, so $d = h/\mu_k$

$v \approx 4.9 \text{ m/s}$. It stops after $1.8/0.20 = 9.0 \text{ m}$, whatever its mass



On the rough floor ($f_k = 4.0 \text{ N}$), $K + U$ falls 4.0 J per meter slid. Add the thermal energy and the total stays flat.

Quick check

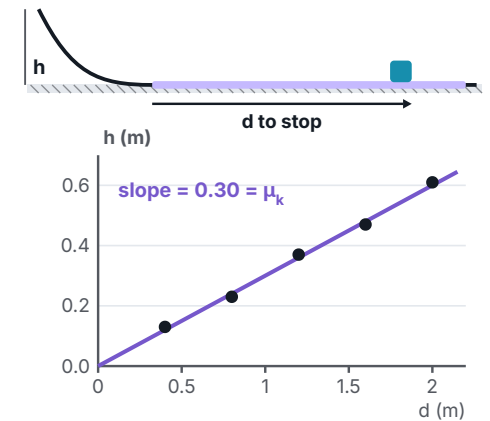
A 0.50 kg ball is dropped from 20 m and lands at 18 m/s. How much energy did air resistance take?

Answer: $mgh = 100 \text{ J}$, $\frac{1}{2}(0.50)(18)^2 = 81 \text{ J}$: 19 J went to thermal energy and sound.

Energy of a swinging rod

A uniform 2.0 kg rod 1.0 m long, pivoted at one end, swings down from horizontal. Its center of mass drops 0.50 m, so U_g falls by $(2.0)(10)(0.50) = 10 \text{ J}$, which becomes the swing's kinetic energy (Topic 6.1).

Energy in the lab question



Release the block from several heights h ; measure how far, d , it slides on the rough floor.

Finding μ_k with only a meterstick

- Change only the release height h . At each height, measure the stopping distance d several times and average.
- The ramp is frictionless, so all of mgh becomes thermal energy on the rough floor: $mgh = \mu_k mgd$, and $h = \mu_k d$.
- Graph h against d and draw a best-fit line. Its slope is μ_k ; no mass or stopwatch needed.

Worked example

A block slides from rest a distance d down a rough 37° ramp to a photogate. A graph of v^2 against d has slope 6.0 m/s^2 . Find μ_k .

- $\frac{1}{2}mv^2 = mgd \sin \theta - \mu_k mg \cos \theta d$
- Slope = $2g(\sin \theta - \mu_k \cos \theta) = 20(0.60 - 0.80\mu_k)$

$6.0 = 12 - 16\mu_k$, so $\mu_k \approx 0.38$

Know by heart

The spring launcher. When a spring launches a ball straight up with no losses, $\frac{1}{2}k(\Delta x)^2 = mgh$, where h runs from the ball's place on the squeezed spring to the top of its flight. A graph of h against $(\Delta x)^2$ has slope $s = k/2mg$, so $k = 2mgs$.

What to measure, and with what

K m with a balance; v with a photogate
 U_g vertical height (meterstick), not ramp length
 U_s k from the slope of force against stretch; Δx from the relaxed length

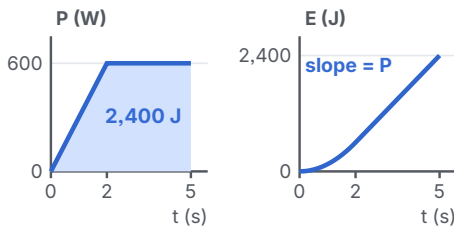
3.5 Power

Power is how fast energy moves into or out of a system, or changes form inside it. It is measured in watts: $1 \text{ W} = 1 \text{ J/s}$.

On the equation sheet

$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{\Delta E}{\Delta t}$$

$$P_{\text{inst}} = F_{\parallel} v = F v \cos \theta$$



The area under the power-time graph is the energy transferred (2,400 J here), and the slope of the energy-time graph is the power.

Worked example

A motor lifts 40 kg up 3.0 m at a steady speed in 6.0 s. What is its power output?

- 1 $W = mg\Delta y = (40)(10)(3.0) = 1,200 \text{ J}$
- 2 $P = W/\Delta t = 1,200/6.0$
- 3 Check with $P = Fv$: $v = 0.50 \text{ m/s}$ and $F = mg = 400 \text{ N}$, so $Fv = 200 \text{ W}$

200 W of mechanical output; the motor draws more electrical power than that

Know by heart

Lifting at a steady speed. The lifting force equals the weight, so $P = Fv$ becomes $P = mgv$.

Measuring your own power on the stairs

- 1 Find your mass on a scale. Measure one step's height with a meterstick and multiply by the number of steps.
- 2 Time the climb with a stopwatch; repeat and average.
- 3 $P = mg\Delta y/\Delta t$. A faster climb does the same work in less time, so the power is greater.

Exam trap

Mixing up watts and joules. Watts measure how fast energy flows; joules measure how much. A 10 W charger running for 2.0 h transfers 72,000 J, and so does a 60 W bulb lit for 20 min. To go from one to the other, use energy = power $\times$ time, with the time in seconds.

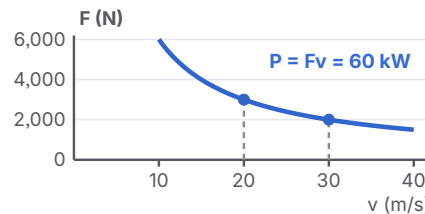
Worked example

A 60 kg student climbs 4.0 m of stairs, once walking (8.0 s) and once running (4.0 s). Compare the work and the power.

- 1 Work both times: $mg\Delta y = (60)(10)(4.0) = 2,400 \text{ J}$
- 2 Walking: $2,400/8.0 = 300 \text{ W}$.
Running: $2,400/4.0 = 600 \text{ W}$

Same work in half the time takes twice the power

Power at one instant: $P = F_{\parallel} v$



An engine at a steady 60 kW pushes with 3,000 N at 20 m/s but only 2,000 N at 30 m/s, because $F = P/v$. That is why a car speeds up more slowly as it goes faster.

Worked example

A rope pulls a box with 50 N at 37° to its motion, and the box moves at 2.0 m/s. How much power does the rope deliver?

- 1 $P = Fv \cos \theta = (50)(2.0)(0.80)$

80 W: only the part of the force along the motion delivers power

Power and the direction of the force

- A force at right angles to the velocity delivers no power: it can't change the speed.
- A force against the motion has negative power: it drains energy.
- At a steady speed against drag, the engine supplies energy as fast as drag removes it. Against 600 N of drag at 25 m/s, that is 15 kW.

Exam trap

Thinking a constant force means constant power. $P = Fv$, so a steady 20 N push delivers 20 W at 1.0 m/s but 60 W at 3.0 m/s. Use Fv at each speed, or $W/\Delta t$ for the average.

Quick check

A 1,200 kg car goes from rest to 20 m/s in 8.0 s on a level road. What average power went into its kinetic energy?

Answer: $\Delta K = \frac{1}{2}(1200)(20)^2 = 240,000 \text{ J}$, and $240,000 \text{ J} \div 8.0 \text{ s} = 30 \text{ kW}$.

Choosing forces or energy

Energy connects the start and end of a motion without needing the time in between. Newton's second law tells you what is happening at one instant.

When the question asks for

Use

a speed at some height, a height, a squeeze or a stopping distance

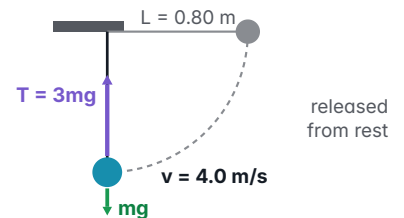
Energy: no time needed

a time, an acceleration, or a force at one instant

Newton's second law and the kinematics equations

a force at the top or bottom of a circle

Both: energy for the speed, then $F_{\text{net}} = mv^2/r$



Worked example

A 0.50 kg bob on a 0.80 m string is let go from rest with the string horizontal. Find the tension at the bottom.

- 1 Energy (the tension does no work, since it pulls at right angles to the motion): $mgL = \frac{1}{2}mv^2$, so $v^2 = 2gL = 16 \text{ m}^2/\text{s}^2$
- 2 At the bottom: $T - mg = mv^2/L$, so $T = 5.0 + \frac{(0.50)(16)}{0.80}$

$T = 15 \text{ N} = 3mg$, for any string length

Exam trap

Expecting energy to give you a time. Energy gives the speed at each point, not when the object gets there. A block sliding 2.5 m down a frictionless 30° ramp drops 1.25 m, so energy gives 5.0 m/s at the bottom. For the time, use $a = g \sin \theta = 5.0 \text{ m/s}^2$ and kinematics: $t = v/a = 1.0 \text{ s}$.

Check the units

$\sqrt{2gh}$ should come out in m/s and $mgh/\Delta t$ in watts. If they don't, the algebra has gone wrong.

② Solving an energy problem, step by step

- 1 Choose the system. With Earth inside, U_g goes in the equation; with the block alone, gravity does work on it from outside instead.
- 2 List its energies at the start and the end: K for anything moving, U_g if Earth is inside, U_s for a spring, thermal energy for a rough surface.
- 3 Check what crosses the boundary. Work done by an outside force goes on the starting side. Energy that friction turns into thermal energy goes on the ending side.
- 4 Write the equation, cross out each term that is zero and say why ("it starts from rest"), then solve.

🔗 What earns points on Unit 3 free-response questions

When the question says	What earns the point
"Draw energy bar charts for the system at A and B"	Bars start at the zero line with heights to scale, and a zero energy is a short line on the zero line. If nothing outside does work, the totals at A and B match. Include only that system's energies.
"Starting with conservation of energy, derive..."	Write $K_i + U_i = K_f + U_f$ (add $f_k d$ if there is friction), cross out the zero terms, use the vertical drop $d \sin \theta$, square the whole Δx , and solve using only the symbols allowed.
"Sketch E , U_g and U_s against position"	E is a horizontal line. U_g is a straight line that falls as the block goes down a straight ramp. U_s is a parabola starting where the spring is first touched. K is the gap, $E - U_g - U_s$.
"Greater than, less than or equal to?" for two speeds	Compare how much each potential energy changes, say how K changes as a result, then the speed. Name the system and explain why its mechanical energy is constant.
"Determine the work done by a force"	Get the sign from the angle between force and motion; use $F_{\parallel} d$ or the area under the force graph. It is zero if the force is at right angles to the motion or the spot it acts on doesn't move.
"Describe a procedure to find μ_k or k "	Say what you measure and with what, change one quantity at a time, repeat trials, and name a graph that comes out straight and what its slope equals.
"Use your best-fit line to calculate..."	Take the slope from two points on your line, not two data points, with its units. Then use the slope relation: $\mu_k = \text{slope for } h \text{ against } d$, and $k = 2mg \times \text{slope for } h \text{ against } (\Delta x)^2$.

Where energy shows up in other units

- 2.9 Circular motion.** Energy gives the speed in a loop or a swing; then $F_{\text{net}} = mv^2/r$ gives the force.
- 4.4 Collisions.** When the net outside impulse is negligible, momentum is conserved but kinetic energy usually isn't. Find the velocities first, then compare K before and after.
- 6.1 Rotation.** $K = \frac{1}{2}I\omega^2$ and $W = \tau\Delta\theta$. A rolling object's kinetic energy is split between moving along and spinning (Topic 6.5). Escape speed comes from U_g (Topic 6.6).
- 6.2**
- 7.4 Oscillations.** A mass on a horizontal spring trades K and U_s . Their sum stays constant, and the mass moves fastest at the equilibrium position.
- 8.4 Fluids.** Bernoulli's equation is conservation of energy for a moving fluid, per unit volume.

📊 Worked example

A spring of constant k , squeezed by Δx , launches a cart of mass M from rest along a level frictionless track and up a frictionless ramp. Derive the height H the cart reaches.

- 1 System: cart, spring and Earth. With no friction and no outside work, $K + U_s + U_g$ stays constant
- 2 At the start the cart is at rest at the bottom; at the top it stops for an instant and the spring is relaxed. So $\frac{1}{2}k(\Delta x)^2 = MgH$

$$H = \frac{k(\Delta x)^2}{2Mg}: \text{twice the squeeze sends the cart four times as high}$$

🔗 What the question's verb asks for

Derive	Start from a law or a given equation, show each step, and end in symbols.
Calculate	Show the steps with numbers put in, and give units and sensible significant figures.
Determine	Reach a value or a conclusion; the work can be brief.
Justify	Give reasons in words that go beyond the algebra: name the system and the principle.
Sketch	Get the shape and trend right, matching your other answers.
Indicate	Pick the answer; no reason needed.

🔗 Sentences that earn points

- "No outside force does work on the block-Earth-spring system and there is no friction, so its mechanical energy is constant."
- "From P to Q the system loses more U_g than it gains U_s , so K increases and the speed at Q is greater."
- "The normal force is perpendicular to the displacement, so it does no work."
- "Kinetic friction turns mechanical energy into thermal energy, so $K + U$ falls by $f_k d$."
- "Both lifts do the same work, $mg\Delta y$, but B takes less time, so B's power is greater."
- "With Earth outside the system, gravity does positive work on the ball, so the ball's kinetic energy increases."
- "The rocket-Earth system's gravitational potential energy increases because the distance between the rocket and Earth increases."

Results worth knowing how to derive

Speed after a drop h	$v = \sqrt{2gh}$
Speed off a spring	$v = \Delta x \sqrt{k/m}$
Sliding to a stop	$d = v^2/(2\mu_k g)$
Staying on a loop	$h \geq 2.5R$
Swing from horizontal	$T_{\text{bottom}} = 3mg$
Lifting at steady speed	$P = mgv$

None is on the equation sheet. In a derivation, start from the principle.