

Unit 4

AP Physics 1

Linear Momentum

Momentum · Impulse · Force–time graphs · Systems · Center of mass · Collisions · Explosions

Unit 1
10–15%
Kinematics

Unit 2
18–23%
Force and
dynamics

Unit 3
18–23%
Work, energy,
power

Unit 4
10–15%
Linear
momentum

Unit 5
10–15%
Torque and
rotation

Unit 6 5–8%
Rotating
systems

Unit 7 5–8%
Oscillations

Unit 8
10–15%
Fluids

Unit 4 is 10–15% of the multiple-choice section. Any of the four free-response questions can use it. Recent exams had a collision in question 1, which asks for calculations, and in question 2, which asks for graphs and diagrams.

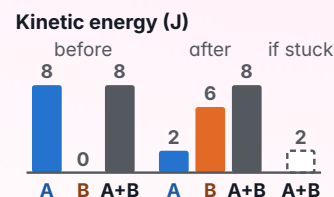
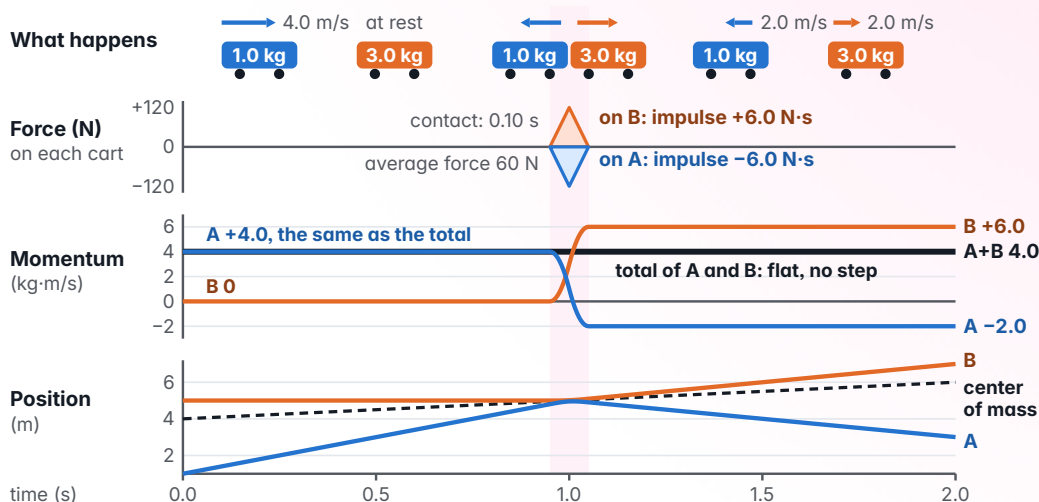
4.1 Linear momentum

4.2 Change in momentum and impulse

4.3 Conservation of linear momentum

4.4 Elastic and inelastic collisions

What happens



Bounce: total K stays 8.0 J. If stuck: both 1.0 m/s, total K 2.0 J. Total p is 4.0 either way.

Cart A (blue, 1.0 kg) at +4.0 m/s hits cart B (orange, 3.0 kg) at rest on a frictionless track. The arrows in the middle picture are the forces the carts exert on each other: equal and opposite for the same 0.10 s, so A loses 6.0 kg·m/s and B gains exactly that. The total stays 4.0 kg·m/s and the center of mass keeps a steady speed. The total kinetic energy is 8.0 J before and after, so the collision is elastic.

What the exam asks most

- The speed after a collision or explosion.** Set the total momentum before equal to the total after, with a term for every object and a sign on each velocity. If a 2.0 kg cart at +3.0 m/s sticks to a 1.0 kg cart at rest, $(2.0)(3.0) + 0 = (3.0)v$, so $v = +2.0$ m/s. (4.3) (4.4)
- A graph of the system's total momentum.** With no outside force, draw one flat line at the starting total, straight through the collision. Each cart's own graph changes sharply during the hit, but any step or bump in the total loses the point. (4.3)
- Whether a collision is elastic.** Use momentum for the speeds after the collision, then add up every object's kinetic energy before and after. Equal totals mean elastic. If the objects then swing up or squeeze a spring, use energy only for that later motion. (4.4)
- An impulse or an average force.** The net impulse equals the change in momentum, or the area under a net force–time graph; divide it by the contact time for the average net force. A bounce flips the sign of the velocity: a 0.20 kg ball that hits a wall at +5.0 m/s and leaves at -4.0 m/s has $\Delta p = 0.20(-4.0 - 5.0) = -1.8$ kg·m/s. (4.2)
- Why the total momentum does or doesn't change.** Name the system, sort the forces into inside and outside, and say whether the outside ones add to zero. For a block sliding on a cart: "The system is the block and the cart. Friction between them is internal, and the track is frictionless, so no outside force acts sideways and the horizontal momentum stays constant." Writing only "momentum is conserved" earns nothing. (4.3)

On the equation sheet

$$\vec{p} = m\vec{v} \quad K = \frac{1}{2}mv^2$$

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$$

$$\vec{F}_{\text{net}} = \frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$$

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i} = \frac{\sum m_i\vec{v}_i}{\sum m_i}$$

The equation sheet lists $g = 9.8$ m/s², but exam questions use 10 m/s², as the examples here do. Using 9.8 or 9.81 correctly also earns full credit.

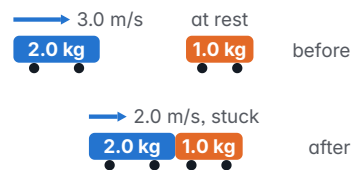
Not on the equation sheet: know these

- Change in momentum is final minus initial, with signs: $\Delta\vec{p} = \vec{p} - \vec{p}_0$.
- With no net outside force, $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$, and two objects that push only on each other have $\Delta\vec{p}_A = -\Delta\vec{p}_B$.
- Impulse is the area under a net force vs time graph; net force is the slope of a momentum vs time graph.

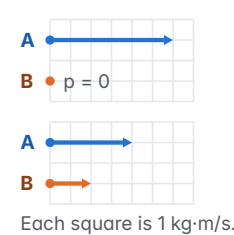
4.1 Linear momentum

Momentum is mass times velocity, and its sign gives its direction

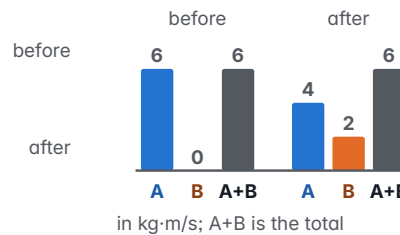
The carts



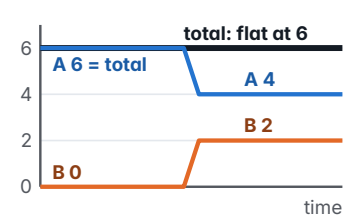
Momentum arrows



Bar chart



Momentum vs time



A 2.0 kg cart at +3.0 m/s hits a 1.0 kg cart at rest, and they stick and roll on together at +2.0 m/s. In every panel the total momentum is 6.0 kg·m/s, before and after. Free-response questions often give you one of these pictures and ask you to draw another.

Momentum is mass times velocity. Mass is never negative, so momentum points the way the object moves. Pick a positive direction, and the sign of each momentum tells you which way it points.

On the equation sheet

$$\vec{p} = m\vec{v} \quad \text{unit: kg}\cdot\text{m/s, the same as N}\cdot\text{s}$$

A system's momentum is the sum of its parts' momenta, $\sum m_i \vec{v}_i$, which also equals its total mass times its center-of-mass velocity.

Worked example

A 2.0 kg cart at +3.0 m/s meets a 3.0 kg cart at -2.0 m/s. Find the system's momentum and kinetic energy.

$$\begin{aligned} 1 \quad p &= (2.0)(+3.0) + (3.0)(-2.0) = +6.0 - 6.0 = 0 \\ 2 \quad K &= \frac{1}{2}(2.0)(3.0)^2 + \frac{1}{2}(3.0)(2.0)^2 = 9.0 + 6.0 = 15 \text{ J} \end{aligned}$$

$$p_{\text{sys}} = 0, \text{ but } K_{\text{sys}} = 15 \text{ J}$$

The momenta cancel because they point opposite ways, but kinetic energy has no direction, so the energies add. If these carts stuck together, they would stop dead and all 15 J would become internal energy.

Exam trap

- **Adding momenta as if they had no direction.** Carts with +6.0 and -6.0 kg·m/s don't have 12 kg·m/s between them. Add with signs: $+6.0 + (-6.0) = 0$. The minus sign doesn't make a momentum smaller; it only says which way it points.
- **Drawing equal momentum arrows because the speeds are equal.** Momentum is mv , so at the same 2.0 m/s a 2.0 kg cart has 4.0 kg·m/s and a 1.0 kg cart has 2.0 kg·m/s. Make the heavier cart's arrow twice as long.

How momentum and kinetic energy change

If you	$p = mv$	$K = \frac{1}{2}mv^2$
double m	$\times 2$	$\times 2$
double v	$\times 2$	$\times 4$
double m , halve v	$\times 1$	$\times \frac{1}{2}$
reverse v	sign flips	no change

Not on the equation sheet: K from p

$$K = \frac{p^2}{2m}$$

This is the sheet's $K = \frac{1}{2}mv^2$ written as $(mv)^2/2m$. So with equal momenta, the lighter object has more kinetic energy; with equal kinetic energies, the heavier object has more momentum.

Before: at rest, total $p = 0$



After the spring pushes them apart



A compressed spring pushes two blocks apart from rest. Their momenta are equal and opposite, so the total stays zero, while the kinetic energy rises from zero to 6.0 J, released from the compressed spring. The lighter block gets three times as much, because $K = p^2/2m$ and both momenta are the same size.

Center-of-mass velocity

Divide a system's momentum by its total mass and you get its center-of-mass velocity. For the carts at the top of the page, $v_{\text{cm}} = \frac{6.0 \text{ kg}\cdot\text{m/s}}{3.0 \text{ kg}} = 2.0 \text{ m/s}$, before the hit and after.

That is exactly the speed of the stuck pair. Objects that stick always move off with the velocity their center of mass had before the hit.

Collisions and explosions

In a **collision**, the objects push on each other far harder than anything outside pushes on them, so for the brief contact you can ignore outside forces and compare just before with just after.

In an **explosion**, forces inside the system push its parts apart. If the system starts at rest and nothing outside pushes, the parts' momenta add to zero.

The exam won't ask you to calculate the motion of a rocket or anything else whose mass changes as it moves.

Finding momentum from a graph

Graph	What the graph tells you
x vs t	$p = m \times \text{the slope}$
v vs t	$p = m \times \text{the value you read}$
p vs t	the slope is the net force (4.2)
F_{net} vs t	the area is the change in momentum (4.2)

How to draw momentum arrows

Each object is shown as a dot. Start its arrow there, point it the way the object moves, and make it mv long on the given scale; write " $p = 0$ " beside a stopped object. Then put the after arrows tip to tail: together they must reach exactly as far as the total before.

Quick check

A 0.40 kg cart's position vs time graph is a straight line from 0 m at $t = 0$ to 1.2 m at $t = 2.0$ s. What is its momentum?

Answer: the slope is $1.2/2.0 = 0.60$ m/s, so $p = (0.40)(0.60) = 0.24$ kg·m/s.

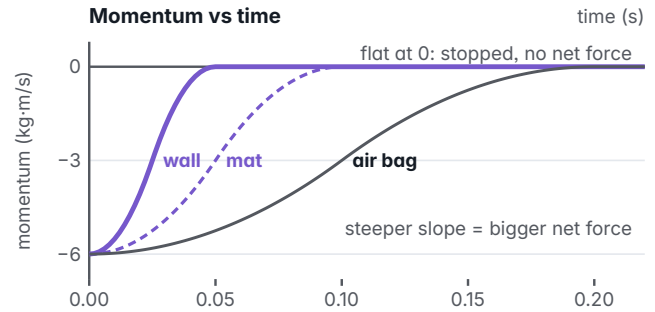
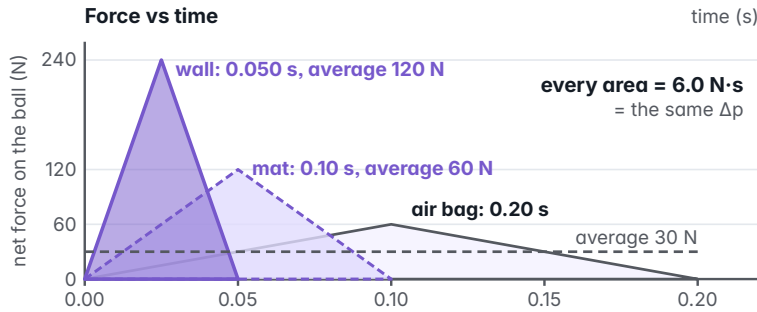
Quick check

Which has more momentum: a 1,200 kg car at 2.0 m/s or a 0.010 kg bullet at 400 m/s?

Answer: the car, 2,400 kg·m/s against 4.0 kg·m/s.

4.2 Change in momentum and impulse

Average net force times time equals the change in momentum



A 0.50 kg ball moving at 12 m/s in the negative direction is stopped by a wall, a mat or an air bag. Each stop changes its momentum by the same $0 - (0.50)(-12) = +6.0 \text{ kg·m/s}$, so the areas are equal, and the longer the stop, the smaller the average force. On the right, the slope of each momentum curve is the net force at that moment.

Net force is the rate at which momentum changes. Average net force times the time it acts is the **net impulse**, and it equals the change in momentum. One force alone gives an impulse of that force times the time.

On the equation sheet

$$\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} = m \vec{a}$$

$$\vec{J} = \vec{F}_{\text{avg}} \Delta t = \Delta \vec{p} \quad \text{N·s = kg·m/s}$$

Over a time interval, the first equation gives the average net force.

Not on the equation sheet: know these

- Change in momentum is final minus initial, with signs: $\Delta \vec{p} = \vec{p} - \vec{p}_0$.
- Impulse is the area under a net force vs time graph.
- Net force is the slope of a momentum vs time graph.
- The area under a force vs position graph is work (Unit 3), not impulse.

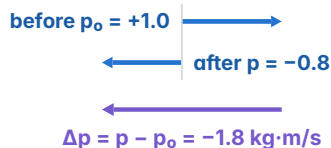
Worked example

A 0.20 kg ball hits a wall at +5.0 m/s and bounces straight back at 4.0 m/s after 0.030 s of contact. Find the impulse on it and the wall's average force.

$$1 \quad \Delta p = m(v - v_0) = 0.20(-4.0 - 5.0) = -1.8 \text{ kg·m/s}$$

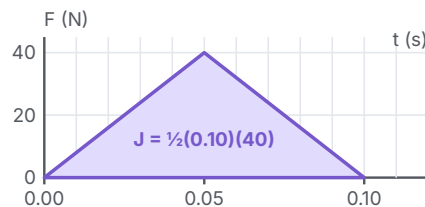
$$2 \quad F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{-1.8}{0.030} = -60 \text{ N}$$

$J = -1.8 \text{ N·s}$; $F_{\text{avg}} = -60 \text{ N}$, away from the wall



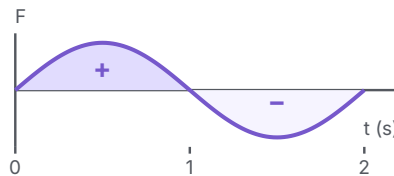
Draw $\vec{p}_0$ and $\vec{p}$ from the same point. $\Delta \vec{p}$ is the arrow from the tip of $\vec{p}_0$ to the tip of $\vec{p}$.

Impulse is the area under the force curve



$J = \frac{1}{2}(0.10 \text{ s})(40 \text{ N}) = 2.0 \text{ N·s}$, so the average force is 20 N, half the peak. On a 0.50 kg cart, $\Delta v = J/m = 4.0 \text{ m/s}$. Spread over 0.20 s, the same impulse needs only 10 N.

For an irregular curve, count the grid squares under it and multiply by one square's impulse (its width in seconds times its height in newtons).



Area below the axis counts as negative. A spring that pushes and then pulls back just as hard for just as long gives zero net impulse, so the cart keeps its starting momentum.

Exam trap

- Subtracting speeds when an object bounces.** $0.20(4.0 - 5.0)$ treats the bounce as a slowdown. The ball reversed, from +5.0 to -4.0 m/s, so $\Delta p = -1.8 \text{ kg·m/s}$, not -0.20.
- Treating the impulse as the final momentum.** A 4.0 N net force acting for 2.0 s gives 8.0 N·s, which is the *change* in momentum. A cart that started with 3.0 kg·m/s ends with 11 kg·m/s, not 8.0.
- Assuming impulse points the way the object moves.** It points the way the momentum *changes*. Taking up as positive, a 60 kg stunt performer landing at 5.0 m/s downward on an air bag has $\Delta p = 0 - (60)(-5.0) = +300 \text{ kg·m/s}$: the impulse is upward, though they were moving down.

Finding the floor's force

Worked example

A 0.40 kg ball hits the floor at 6.0 m/s and leaves at 4.0 m/s upward after 0.020 s of contact. Find the floor's average force on it.

- Up is +: $\Delta p = 0.40[(+4.0) - (-6.0)] = +4.0 \text{ kg·m/s}$
- $F_{\text{net}} = 4.0/0.020 = +200 \text{ N}$
- $F_{\text{floor}} - mg = 200$, so $F_{\text{floor}} = 200 + (0.40)(10)$

$F_{\text{floor}} = 204 \text{ N}$, or $2.0 \times 10^2 \text{ N}$ upward to two significant figures

$\Delta p/\Delta t$ gives the average **net** force, the floor's push minus the weight. The floor's own impulse is $(204)(0.020) = 4.08 \text{ N·s}$ and gravity's is -0.08 N·s ; together they make $\Delta p = +4.0 \text{ kg·m/s}$. Leave out the weight only if the question says to.

How to explain a softer landing

An egg dropped on the floor and one dropped on a pillow both go from the same speed to rest, so Δp and the impulse are the same. The pillow makes Δt longer, so $F_{\text{avg}} = \Delta p/\Delta t$ is smaller, and the egg doesn't break.

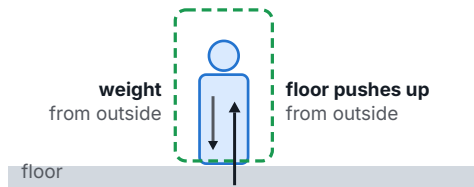
Lab: measuring a mass with impulse

Push a cart along a level track with negligible friction using a handheld force sensor, so the sensor reads the net force, while a motion detector records the velocity before and after. Do at least five different pushes. Graph impulse (the area under each force curve) on the vertical axis against Δv : since $J = m\Delta v$, the best-fit line's slope is the mass. Take the slope from two far-apart points on your line, not from data points; page 6 shows a calculator check.

4.3 Conservation of linear momentum

Only forces from outside a system can change its total momentum

System: the jumper



The jumper's momentum changes.

Forces between objects inside the dashed line are internal: they come in third-law pairs whose impulses cancel. Only forces from objects outside the line can change the system's total. With Earth inside, the floor's push and Earth's pull on the jumper are both internal, so the jumper-and-Earth total stays zero.

Objects in a system can trade momentum, but the total changes only when something outside the system pushes or pulls.

Not on the equation sheet:
conservation of momentum

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$$

This holds when the net outside force is zero. For two objects (primes mark the velocities after):

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

If an outside force does act, the system's change in momentum equals the outside impulse: $\Delta \vec{p}_{\text{sys}} = \vec{J}_{\text{ext}}$.

Worked example

Two skaters, 50 kg and 75 kg, push apart from rest. The 75 kg skater moves off at +2.0 m/s. Find the other skater's velocity.

$$1 \quad 0 = (50)v + (75)(+2.0)$$

$$2 \quad v = -150/50$$

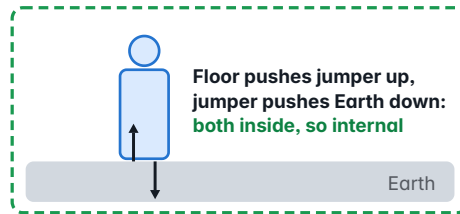
$$v = -3.0 \text{ m/s, so } 3.0 \text{ m/s the opposite way}$$

Their kinetic energy went from 0 to $225 + 150 = 375 \text{ J}$, supplied by their muscles. A push-apart always adds kinetic energy.

Exam trap

- Drawing a step in the system's momentum graph. Each cart's momentum changes at the collision, but the changes are equal and opposite: on page 1, A goes from +4.0 to -2.0 kg·m/s and B from 0 to +6.0. The total stays +4.0, one flat line.
- Calling the friction between a block and its cart external. If both are in your system, the friction on the block and the friction on the cart are a third-law pair inside it, so their impulses add to zero in the system's total. Only a force from outside, such as friction from the track, is external.

System: jumper + Earth



The total stays 0; Earth's recoil is tiny.

Momentum kept in one direction only

Worked example

A 1.0 kg block is released from rest just above a 4.0 kg cart rolling at 1.5 m/s on a frictionless track. It lands on the cart and sticks. Find the new speed and the change in kinetic energy, counting only the horizontal motion.

$$1 \quad \text{Horizontal: } (4.0)(1.5) + (1.0)(0) = (5.0)v_f$$

$$2 \quad v_f = 6.0/5.0 = 1.2 \text{ m/s}$$

$$3 \quad K: \frac{1}{2}(4.0)(1.5)^2 = 4.5 \text{ J} \rightarrow \frac{1}{2}(5.0)(1.2)^2 = 3.6 \text{ J}$$

$$v_f = 1.2 \text{ m/s; } \Delta K = -0.90 \text{ J}$$

Vertical momentum isn't conserved: the track pushes up on the cart, from outside the system, to stop the block's fall. Horizontal momentum is, since nothing outside pushes sideways. The block starts with no horizontal velocity, so its term before is 0.

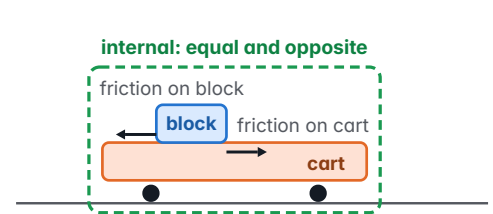
When the momentum stays constant

Event	System	Is $\vec{p}$ constant?
A ball falls	the ball	no, Earth's gravity is outside
A ball falls	ball and Earth	yes, gravity is now inside
Carts collide on a frictionless track	both carts	yes, along the track
A dart hits a hanging block	dart and block	yes, horizontally, during the hit

Why the two changes are equal

When A and B push only on each other, $\Delta \vec{p}_A = -\Delta \vec{p}_B$: the forces they exert on each other are equal and opposite (Newton's third law) and act for the same time, so their impulses are equal and opposite.

System: block + cart



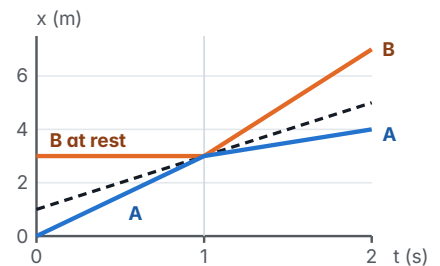
Horizontal momentum stays constant.

Center-of-mass motion

On the equation sheet

$$\vec{v}_{\text{cm}} = \frac{\sum \vec{p}_i}{\sum m_i}$$

With no net outside force, the total momentum (the top of the fraction) and the total mass both stay constant, so $\vec{v}_{\text{cm}}$ does too, even through a collision.

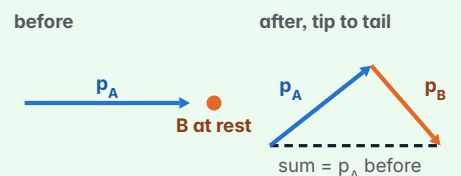


Cart A (2.0 kg, 3.0 m/s) hits cart B (1.0 kg, at rest) elastically; afterward A moves at 1.0 m/s and B at 4.0 m/s. The center of mass (dashed) moves at 2.0 m/s throughout.

How to sketch positions after a collision

- 1 Draw the center of mass as one straight line that keeps its slope through the hit.
- 2 Draw each object as a straight line from the collision point, with a slope equal to its velocity after the hit.
- 3 Keep the center of mass between them, closer to the heavier object.

Collisions in two dimensions



Momentum is conserved along x and along y separately. Placed tip to tail, the arrows after must add up to A's arrow before, so their y parts cancel. You may be asked to write the x and y equations, but not to solve them.

4.4 Elastic and inelastic collisions

Every collision keeps the total momentum; kinetic energy shows which kind it is

Before, in every row

	total $p = +6.0 \text{ kg}\cdot\text{m/s}$	total $K = \frac{1}{2}(1.0)(6.0)^2 = 18 \text{ J}$	
After	Momentum after (kg·m/s)	Kinetic energy after (J)	Type of collision
			Elastic total $K: 18 \rightarrow 18 \text{ J}$ the 2.0 kg cart gets +8
			Inelastic total $K: 18 \rightarrow 9 \text{ J}$ 9 J to internal energy
			Perfectly inelastic total $K: 18 \rightarrow 6 \text{ J}$ they stick together

dashed: the total before

dashed: 18 J before

A 1.0 kg cart (blue) at 6.0 m/s hits a 2.0 kg cart (orange) at rest; the grey bars are the total of both. All three rows end with a total of +6.0 kg·m/s, so momentum alone can't tell you which happened; kinetic energy can.

Not on the equation sheet

- Elastic** The total kinetic energy is the same after as before.
- Inelastic** The total kinetic energy drops; the lost kinetic energy becomes internal energy.
- Perfectly inelastic** The objects stick together and share one velocity.
- Momentum is conserved in all three when outside forces are negligible.

Speed and energy after objects stick

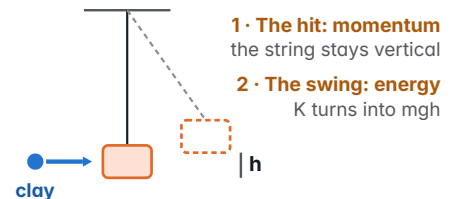
Worked example

A 2.0 kg cart at +3.0 m/s hits and sticks to a 1.0 kg cart at rest. Find v_f and ΔK .

- $(2.0)(3.0) = (3.0)v_f$, so $v_f = 2.0 \text{ m/s}$
- $K_i = \frac{1}{2}(2.0)(3.0)^2 = 9.0 \text{ J}$
- $K_f = \frac{1}{2}(3.0)(2.0)^2 = 6.0 \text{ J}$

$$v_f = 2.0 \text{ m/s}; \Delta K = -3.0 \text{ J}$$

A collision followed by a swing



During the brief hit the string is still vertical, so it can't pull sideways, and horizontal momentum is conserved. During the swing, kinetic energy turns into gravitational potential energy, mgh .

How to show a collision is elastic

The collision is elastic because the system's total kinetic energy after, 18 J, equals the total before, 18 J.

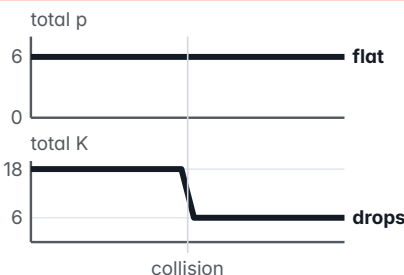
Work out both totals and compare them. "It bounced" doesn't show it, since most bounces lose some kinetic energy, and "momentum was conserved" is true of every collision.

Exam trap

- Thinking each object keeps its own kinetic energy in an elastic collision. Only the total is kept: in the top row, one cart drops from 18 J to 2 J while the other gains 16 J.
- Thinking objects that stick must stop. They stop only if the total momentum was zero. Otherwise they move off together at v_{cm} : in the bottom row, $6.0 \text{ kg}\cdot\text{m/s} \div 3.0 \text{ kg} = 2.0 \text{ m/s}$.

Exam trap

- Using energy conservation to find the speed after objects stick. Setting $\frac{1}{2}(2.0)(3.0)^2 = \frac{1}{2}(3.0)v^2$ gives 2.45 m/s, which is wrong: kinetic energy isn't conserved when objects stick. Find the speed from momentum first.
- Using the combined mass before the hit. $(3.0)(3.0) = (3.0)v$ gives 3.0 m/s, as if the 1.0 kg cart had been moving too. Before the hit, write $(2.0)(3.0) + (1.0)(0)$.
- Writing ΔK as $\frac{1}{2}m(\Delta v)^2$. Kinetic energy depends on v^2 , and squaring a difference isn't subtracting squares: $\frac{1}{2}(3.0)(2.0 - 3.0)^2 = +1.5 \text{ J}$, but $\Delta K = 6.0 - 9.0 = -3.0 \text{ J}$.



For the bottom row above, kinetic energy drops from 18 J to 6 J as sticking turns 12 J into internal energy, while momentum stays at 6.0 kg·m/s.

Worked example

0.050 kg of clay at 12 m/s sticks to a 0.25 kg hanging block. How high do they swing?

- Hit: $(0.050)(12) = (0.30)v$, so $v = 2.0 \text{ m/s}$
- Swing: $\frac{1}{2}(0.30)(2.0)^2 = (0.30)(10)h$

$$h = 0.20 \text{ m}$$

Exam trap

Using energy conservation from the start. Setting the clay's 3.6 J equal to $(0.30)(10)h$ gives $h = 1.2 \text{ m}$, six times too high. The sticky hit turns 3.0 J into internal energy, so use momentum for the hit and energy only for the swing.

Quick check

A 1.0 kg cart at +5.0 m/s hits a 4.0 kg cart at rest and rebounds at 3.0 m/s. Find the 4.0 kg cart's velocity. Is the collision elastic?

Answer: $5.0 = -3.0 + 4.0v$, so $v = +2.0 \text{ m/s}$. K is 12.5 J before and $4.5 + 8.0 = 12.5 \text{ J}$ after, so it is elastic.

Elastic head-on hits on a target at rest

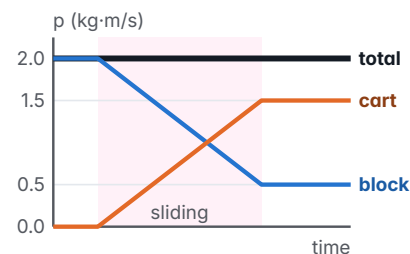
- Equal masses** They swap velocities.
- Heavy hits light** Both move forward.
- Light hits heavy** The light one bounces back.

Free-response questions on momentum

⊙ A block slides onto a cart

A 0.50 kg block slides at 4.0 m/s onto a 1.5 kg cart at rest on a level track with negligible friction. Friction between the block and the cart slows the block and speeds up the cart until they move together.

The part	An answer that earns the point	What earns it, and what loses it
Derive the final speed. 4.3	$(0.50)(4.0) + (1.5)(0) = (2.0)v_f$, so $v_f = 1.0$ m/s	Start from conservation of momentum. Before, each object has its own term; afterward they move as one 2.0 kg object. Starting from energy loses the point.
Find the change in kinetic energy. 4.4	$K_i = \frac{1}{2}(0.50)(4.0)^2 = 4.0$ J and $K_f = \frac{1}{2}(2.0)(1.0)^2 = 1.0$ J, so $\Delta K = -3.0$ J	Work out both totals, using the speed from the first part. The missing 3.0 J became internal energy: the surfaces warmed as they rubbed.
Sketch the system's momentum while the block slides. 4.3	A flat line at 2.0 kg·m/s from start to finish.	A drop or a step during the sliding loses the point.
Sketch each object's momentum. 4.3	The block's momentum falls from 2.0 to 0.50 kg·m/s while the cart's rises from 0 to 1.5 kg·m/s.	The two changes must be equal and opposite, so the lines add to 2.0 at every moment. Both are straight, because the friction force is constant.
Explain why the system's momentum stays constant. 4.3	"The friction forces between the block and the cart are internal: equal and opposite, for the same time, so their impulses cancel. The track exerts no horizontal force, so the system's horizontal momentum stays constant."	Name the forces in this situation and say whether each is inside or outside the system. "Momentum is conserved" on its own earns nothing.



The momenta of the block and the cart change by equal and opposite amounts, 1.5 kg·m/s each, so the total stays at 2.0 kg·m/s.

📱 Check a best-fit slope on a calculator

Ti-84 Plus CE. **[stat]** 1: Edit. Enter Δv in L1 and the impulses in L2. Then **[stat]** → CALC → 4: LinReg(ax+b), set Xlist L1 and Ylist L2, and choose Calculate. The slope is a.

Ti-Nspire CX. On a Lists & Spreadsheet page, enter the data in columns named Δv and j . **[menu]** → Statistics → Stat Calculations → Linear Regression (mx+b), with X List Δv and Y List j . The slope is m.

Desmos. Add a table (+ → table) with Δv in x_1 and the impulses in y_1 , then type $y_1 \sim mx_1 + b$. Desmos shows the slope m.

🔗 Other parts you're likely to see

When the question asks you to	Write or draw this
Draw the momentum arrows after the collision 4.1	Start each arrow at its object's dot, make it the right length on the given scale, and write " $p = 0$ " for a stopped object. The arrows after must add up to the arrows before.
Draw position vs time after the collision 4.3	Each object's line starts at the collision point with its new slope. The center-of-mass line keeps its slope, whether the objects bounce or stick.
Estimate the change in momentum from a force-time graph 4.2	Count the squares under the net-force curve, multiply by one square's impulse, and subtract any area below the axis. Say that this total is the net impulse, which equals Δp .
Compare $ \Delta \vec{p}_A $ and $ \Delta \vec{p}_B $ 4.3	They are equal when A and B push only on each other: the forces are equal and opposite and act for the same time.
Say whether the collision is elastic 4.4	Calculate the total kinetic energy before and after, then compare the two numbers.
Say how v_f changes if a mass doubles 4.4	Put the new mass into the equation you derived, then say whether the result agrees with the question's claim. Above, doubling the cart to 3.0 kg gives $(0.50)(4.0) = (3.5)v_f$, so 0.57 m/s: not half of 1.0 m/s, because the total mass goes from 2.0 kg to 3.5 kg, not 4.0 kg.
Design an experiment 4.2	Measure masses with a balance, speeds with a motion sensor or video, and forces with a force sensor. Repeat with at least five different pushes and graph impulse against Δv ; the slope of the best-fit line is the mass.

🔗 Which idea to use

Objects hit, stick or push apart

A force acts for a known time

A height, a spring, or a force over a distance

A hit, then a swing or a slide

Momentum: the total stays the same.

Impulse: average net force $\times \Delta t = \Delta p$.

Energy: net work $W = \Delta K$, and potential energy turns into K.

Both: momentum for the hit, then energy.

🔗 How Unit 4 connects to other units

Unit 2

Newton's third law is why the impulses in a collision are equal and opposite, and $F = ma$ is $\Delta p / \Delta t$ when the mass stays the same.

Unit 3

The area under force vs position is work; under force vs time, it is impulse.

Unit 6

Angular momentum, $L = I\omega$, follows the same rules, with torque in place of force: $\Delta L = \tau \Delta t$.