

Unit 8

AP Physics 1

Fluids

Density · Pressure at depth · Buoyancy · Floating · Flow rate · Continuity · Bernoulli's equation

Unit 1
10–15%
Kinematics

Unit 2
18–23%
Force and dynamics

Unit 3
18–23%
Work, energy, power

Unit 4
10–15%
Linear momentum

Unit 5
10–15%
Torque and rotation

Unit 6 5–8%
Rotating systems

Unit 7 5–8%
Oscillations

Unit 8
10–15%
Fluids

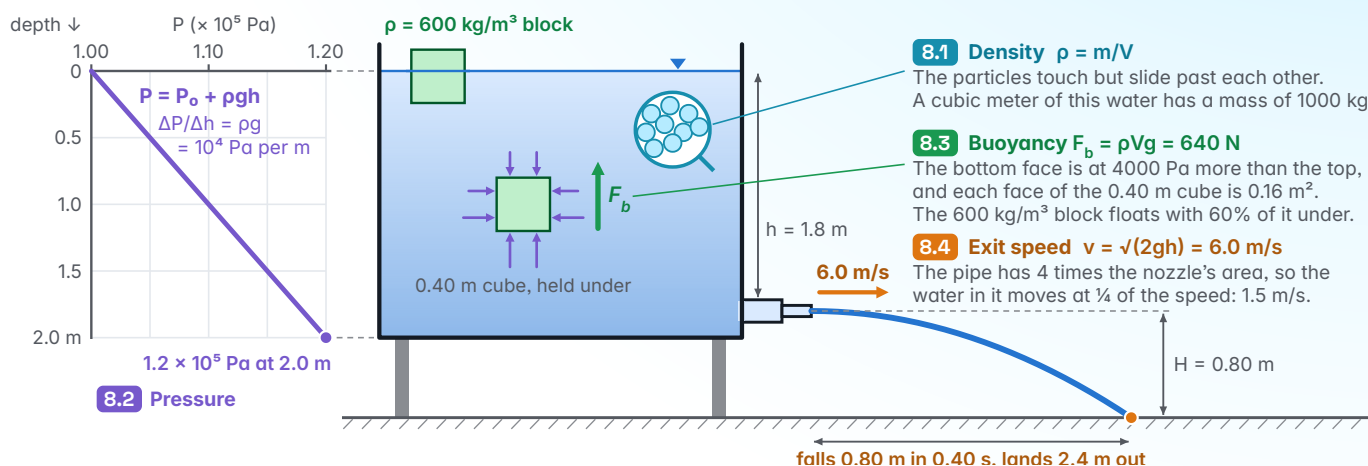
Unit 8 is 10–15% of the multiple-choice section. Any of the four free-response questions can be about fluids. Recent ones compared how fast the same block rises through two different liquids, and asked for a fountain's launch speed and flow rate. You may use a calculator on the whole exam.

8.1 Internal structure and density

8.2 Pressure

8.3 Fluids and Newton's laws

8.4 Fluids and conservation laws



What the exam asks most

- How the same object moves in two liquids.** State that its weight mg is the same in both liquids. The buoyant force ρVg is larger in the denser one, because the object is fully submerged in both (same V) and ρ is larger. So the net upward force and the upward acceleration are larger there. **8.3**
- An expression derived from Newton's second law.** Write $\Sigma F = ma$ first, then put in the forces: $\rho Vg - mg = ma$, so $a = \rho Vg/m - g$. To show it agrees with your claim, point out that ρ is in the positive term $\rho Vg/m$, so a denser liquid gives a larger a . **8.3**
- The force from a pressure.** For a 0.010 m² hatch 2.0 m under water, air presses equally on the water's surface and on the hatch's dry side, so only ρgh counts: $(2.0 \times 10^4 \text{ Pa})(0.010 \text{ m}^2) = 200 \text{ N}$. **8.2**

- Speed and pressure where a pipe narrows.** The same volume passes each point every second, so $A_1v_1 = A_2v_2$: half the radius gives a quarter of the area and four times the speed. At the same height, the faster water has the lower pressure. **8.4**
- Water leaving a hole or a fountain.** Water leaves a hole a depth h below an open surface at $v = \sqrt{2gh}$, then flies as a projectile. A fountain's flow rate is the nozzle's area times the launch speed v_0 , not its vertical part. **8.4**
- Density or g from a graph.** Pressure against depth has slope ρg , so divide the slope by g to get the density. For water jets, the exit speed squared against the hole's depth has slope $2g$. **8.2** **8.4**

On the equation sheet

$$\rho = \frac{m}{V} \quad P = \frac{F_{\perp}}{A}$$

$$P = P_0 + \rho gh \quad P_{\text{gauge}} = \rho gh$$

$$F_b = \rho Vg \quad A_1v_1 = A_2v_2$$

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

$F_{\perp}$ is the force straight into the surface; y is height. The sheet also gives $1 \text{ atm} = 1.0 \times 10^5 \text{ Pa}$ and $A = \pi r^2$, and assumes ideal fluids and completely filled pipes unless a question says otherwise. It lists $g = 9.8 \text{ m/s}^2$, but exam questions are written for $g = 10 \text{ m/s}^2$, as the examples here are; 9.8 or 9.81 also earns full credit.

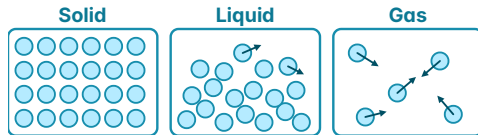
Not on the equation sheet

- Flow rate** (volume per second): $\Delta V/\Delta t = Av$, in m³/s.
- Exit speed** from a hole h below an open surface: $v = \sqrt{2gh}$.
- Floating at rest:** $F_b = mg$, and the fraction of its volume under the surface is $\rho_{\text{obj}}/\rho_{\text{fluid}}$.
- Net force from two pressures** on an area A : $(P_1 - P_2)A$, pushing toward the lower pressure.

8.1 Internal structure and density

Liquids and gases both flow; density is mass per volume

Each particle of a solid is held near a fixed spot, so a solid keeps its shape. In a liquid the particles touch but slide past each other; in a gas they are far apart. Liquids and gases both flow, so both are **fluids**.



What an ideal fluid is

An ideal fluid is **incompressible**: squeezing it doesn't change its volume or density. It has **no viscosity** (no internal friction), so it flows without losing energy. On the exam, treat every fluid as ideal unless the question says otherwise.

Exam trap

Thinking the heavier object sinks. A 300 kg pine log (about 500 kg/m³) floats and a 1 g pebble (about 2700 kg/m³) sinks, because floating depends on density compared with the water's, not on weight.

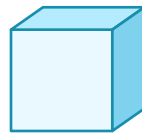
Density is mass per volume

$$\rho = \frac{m}{V}$$

on the equation sheet

Measured in kg/m³, it tells you how tightly the mass is packed.

Quantity	What it means	Unit
mass m	how much matter	kg
volume V	space taken up	m ³
weight mg	Earth's pull on it	N
density ρ	mass per volume	kg/m ³

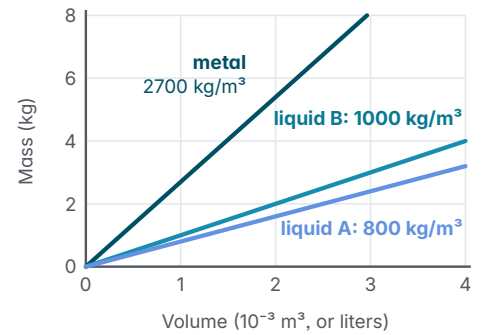


1 m = 100 cm

$$\begin{aligned} 1 \text{ m}^3 &= (100 \text{ cm})^3 \\ &= 10^6 \text{ cm}^3 = 1000 \text{ L} \\ 1 \text{ cm}^3 &= 10^{-6} \text{ m}^3 = 1 \text{ mL} \\ 1 \text{ g/cm}^3 &= 1000 \text{ kg/m}^3 \end{aligned}$$

Exam trap

Converting cm³ to m³ by multiplying by 10⁻². A cubic centimeter is a cube 10⁻² m on each side, so its volume is (10⁻² m)³ = 10⁻⁶ m³. So 20 cm³ is 2.0 × 10⁻⁵ m³, not 0.20 m³.



On a graph of mass against volume, the slope is the density, so a steeper line means a denser material. With the axes swapped, the slope is 1/ρ.

Worked example

A 54 g stone is dropped into a graduated cylinder, and the water rises from 40.0 mL to 60.0 mL. Find the stone's density.

1 The water level rises by the stone's volume: $V = 20.0 \text{ mL} = 20.0 \text{ cm}^3 = 2.00 \times 10^{-5} \text{ m}^3$

2 $\rho = \frac{0.054 \text{ kg}}{2.00 \times 10^{-5} \text{ m}^3}$

$$\rho = 2.7 \times 10^3 \text{ kg/m}^3 \text{ (2.7 g/cm}^3\text{)}$$

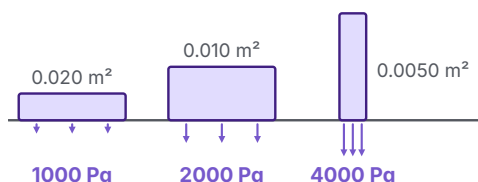
8.2 Pressure

The same force on a smaller area makes a larger pressure

Pressure is the force pushing **straight into** a surface ($F_{\perp}$) divided by its area; a push along the surface makes no pressure. 1 Pa = 1 N/m², and the air around you pushes with about 1.0 × 10⁵ Pa (1 atm).

$$P = \frac{F_{\perp}}{A}$$

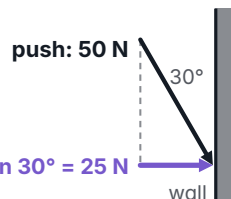
on the equation sheet



The same 20 N brick rests on each of its three faces in turn. On a quarter of the area it makes four times the pressure, which is why a sharp knife cuts and snowshoes keep you on top of the snow.

Pressure is a scalar

Pressure itself has no direction. The force it makes on a surface always pushes **straight into** that surface, whichever way the surface faces. A fluid's pressure comes from its particles hitting the surface, over and over.



$$F_{\perp} = 50 \sin 30^\circ = 25 \text{ N}$$

Worked example

You push a wall with 50 N at 30° to its surface, through a 100 cm² pad that spreads the push evenly. Find the pressure.

1 $F_{\perp} = 50 \sin 30^\circ = 25 \text{ N}$

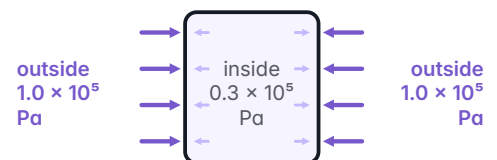
2 $A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2 = 0.010 \text{ m}^2$

$$P = 25/0.010 = 2.5 \times 10^3 \text{ Pa}$$

Quick check

A 60 kg person lies on one nail with a 1.0 mm² tip, then evenly on 1000 such nails. Find the pressure under one tip in each case.

Answer: the weight is 600 N and 1.0 mm² = 1.0 × 10⁻⁶ m², so one nail gives 600/10⁻⁶ = 6.0 × 10⁸ Pa. On 1000 nails each tip carries 0.60 N, so 0.60/10⁻⁶ = 6.0 × 10⁵ Pa.



Pump most of the air out of a sealed can and the outside air crushes it. Nothing inside pulls the walls in; the outside air pushes harder, and the difference is 0.7 × 10⁵ Pa, inward.

$$F_{\text{net}} = (P_{\text{out}} - P_{\text{in}}) A$$

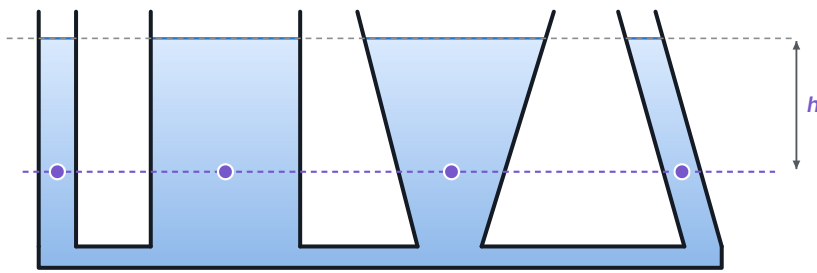
The air on each side of the wall pushes with its own pressure times the area, so the net force points from high pressure to low. On a 0.010 m² panel of the can: (0.7 × 10⁵ Pa)(0.010 m²) = 700 N inward.

Exam trap

Calling a pressure a force. Pressure is force per area, in pascals, so a pressure is never the answer when a question asks for a force. Multiply by an area: 8.0 × 10³ Pa on a 0.050 m² window is 400 N.

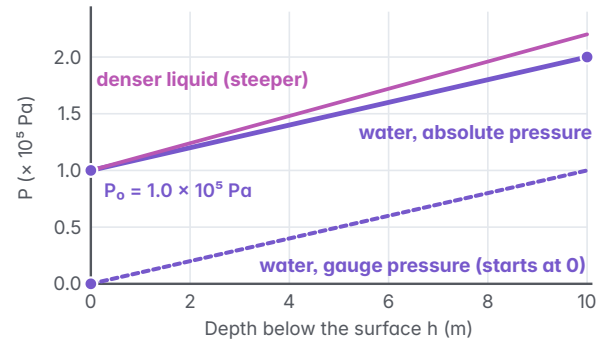
You don't feel the air's 10⁵ Pa because the fluids in your body push outward just as hard, so the net force on your skin is about zero.

8.2 Pressure at depth

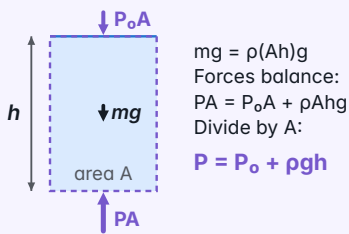
In water, the pressure rises by 10^4 Pa for every meter of depth

The four dots are at the same depth, so they have the same pressure.

Darker blue means higher pressure. In any connected liquid at rest, the pressure depends only on depth, whatever the container's shape. In 10 m of water the pressure doubles, from 1 atm to 2 atm.



Where the equation comes from



Picture a column of liquid at rest. The push from below holds up the column's weight plus the push on its top. The area A cancels, so the pressure depends only on the depth, not on how wide the liquid is.

$$P = P_0 + \rho gh$$

on the equation sheet

$$P_{\text{gauge}} = \rho gh$$

on the equation sheet

Absolute and gauge pressure

– **Absolute pressure** is the total pressure, including the air's; **gauge pressure** is how much more it is than the air's pressure. In an open container, $\text{absolute} = P_0 + \text{gauge}$, where $P_0 = 1.0 \times 10^5$ Pa.

– A tire gauge reads gauge pressure: a flat tire reads zero, though the air inside is still at 1 atm.

– Double the depth and the gauge pressure doubles, but the absolute pressure doesn't: 3.0×10^4 Pa gauge becomes 6.0×10^4 Pa, while the absolute goes from 1.3×10^5 to only 1.6×10^5 Pa.

Why a straw works

Sucking lowers the air pressure in your mouth, and the air pressing on the drink's surface pushes the drink up the straw. Even a perfect vacuum lifts water only about 10 m, where ρgh equals the air's 1.0×10^5 Pa.

Worked example

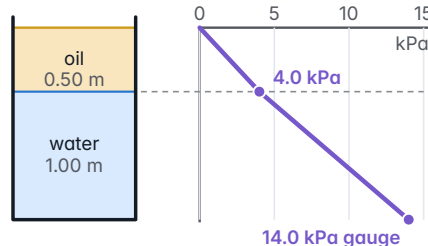
A 0.010 m^2 horizontal hatch is 2.0 m below the surface of an open water tank ($\rho = 1000 \text{ kg/m}^3$), with air at 1.0×10^5 Pa on its dry side. Find the pressure on the wet side and the net force.

- 1 Gauge: $\rho gh = 1000(10)(2.0) = 2.0 \times 10^4$ Pa
- 2 Absolute: $1.0 \times 10^5 + 2.0 \times 10^4 = 1.2 \times 10^5$ Pa
- 3 The air's 1.0×10^5 Pa pushes on both sides and cancels, so the net force is $(2.0 \times 10^4)(0.010)$

$F_{\text{net}} = 200 \text{ N}$, not the 1200 N the absolute pressure gives

Worked example

Oil (800 kg/m^3) 0.50 m deep floats on 1.00 m of water in an open tank. Find the gauge pressure at the bottom.

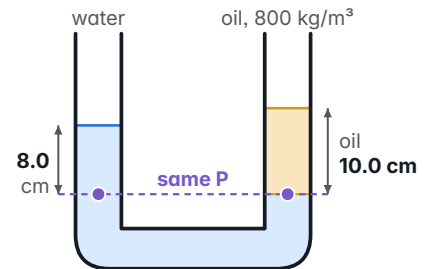


- 1 The oil adds $800(10)(0.50) = 4.0 \times 10^3$ Pa
- 2 The water adds $1000(10)(1.00) = 1.0 \times 10^4$ Pa

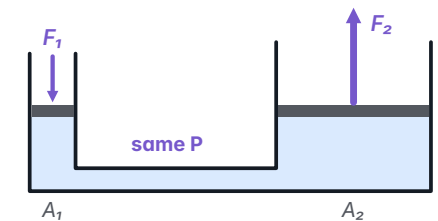
1.4×10^4 Pa gauge (1.14×10^5 Pa absolute)

Each layer adds its own ρgh . The graph bends at the boundary because the slope ρg changes there.

In a closed tank, P_0 is whatever presses on the surface, such as trapped gas or a piston. It is 1 atm only when the surface is open to the air.



Oil floats on the water in the right arm. Level with the oil–water boundary, the two arms' pressures match, so $\rho_{\text{oil}} h_{\text{oil}} = \rho_{\text{water}} h_{\text{water}}$: $800(10.0) = 1000(8.0)$.



In a hydraulic lift both pistons are at the same height in the same fluid, so $F_1/A_1 = F_2/A_2$: 200 N on 0.010 m^2 holds up 10,000 N on 0.50 m^2 . The small piston moves 50 times farther, so work in equals work out. (Not to scale.)

Exam trap

Thinking a wider or longer body of water presses harder. Pressure depends only on depth. A dam is thick because the water behind it is deep, not because the lake is long.

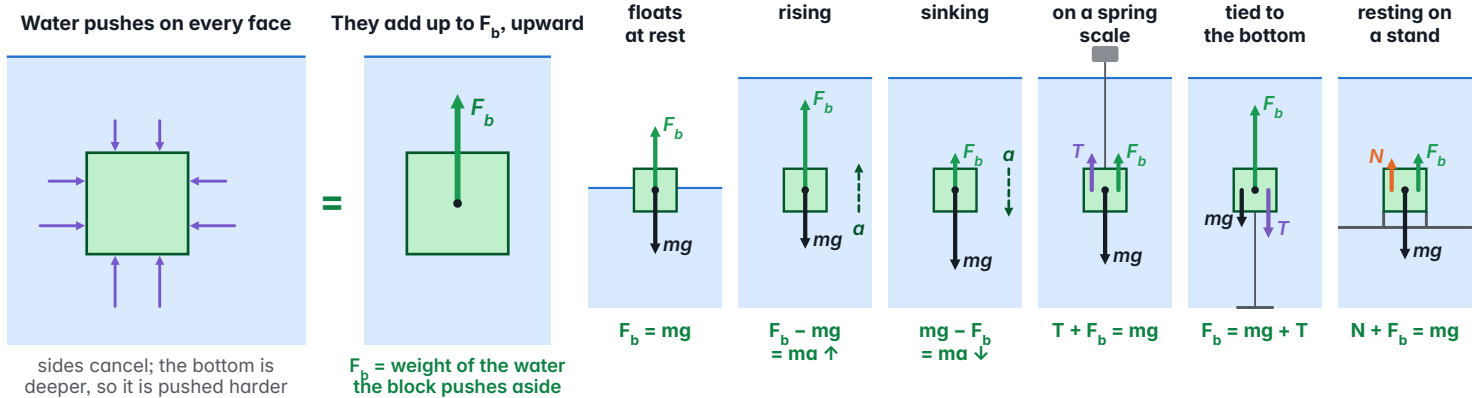
Measuring h along a slanted tube. h is the vertical distance below the surface. A 2.0 m tube running down from the surface at 30° to the horizontal reaches only 1.0 m deep, where the gauge pressure is $1000(10)(1.0) = 1.0 \times 10^4$ Pa.

Sketching pressure against depth

With depth across and pressure up, draw a straight line of slope ρg , starting at P_0 for absolute pressure or 0 for gauge.

8.3 Buoyancy and Newton's laws

The buoyant force is one more force on the free-body diagram



The pressure arrows on the left show only the water's share; the air's share is the same on every face and cancels. In the six diagrams, solid arrows are forces and dashed arrows show the acceleration a . The forces add up to ma , with $a = 0$ at rest.

$$F_b = \rho V g$$

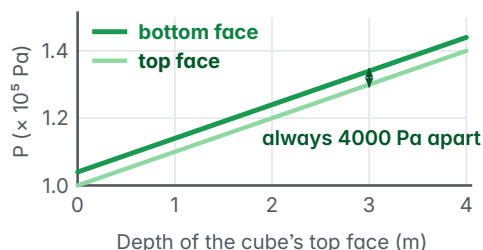
on the equation sheet

ρ density of the **fluid**, not of the object
 V volume of fluid pushed aside: the **submerged part**

Archimedes' principle says the buoyant force equals the weight of the fluid the object pushes aside. To see why, imagine the object replaced by water of the same shape. That water would just stay put, so the water around it must push up with exactly its weight, and it pushes on the object in exactly the same way.

What the buoyant force depends on

- It depends on the fluid's density and on the volume under the surface.
- It doesn't depend on the object's mass or material. Equal-volume lead and aluminum blocks, both fully under water, feel the same buoyant force.
- Once a rigid object is fully under water, the buoyant force doesn't depend on depth either.
- A scuba diver isn't rigid. Deeper down, the water squeezes the air in the diver's lungs and suit, so the volume and the buoyant force shrink and the diver sinks even without swimming down.



Lower a 0.40 m cube and the pressure on both faces rises, but the difference between them stays $\rho g(0.40 \text{ m}) = 4000 \text{ Pa}$, so F_b stays the same.

Deriving the acceleration of a block released under water

Newton's second law $\Sigma F = ma$
 (up is +)
 the two forces $F_b - mg = ma$
 put in $F_b = \rho V g$ $\rho V g - mg = ma$
 solve for a $a = \frac{\rho V g}{m} - g$

Does it agree with the claim that a denser liquid gives a larger acceleration? Yes: ρ is in the numerator of the positive term, so with m and V fixed, a larger ρ gives a larger a . It is not proportional, though: because of the $-g$, a changes by a different factor than ρ . In the example below, 20% more density gives 50% more acceleration.

Worked example

A 0.60 kg block of volume $1.0 \times 10^{-3} \text{ m}^3$ is released from rest fully under water (1000 kg/m^3), then under salt water (1200 kg/m^3). Find each starting acceleration.

- Water: $F_b = 1000(1.0 \times 10^{-3})(10) = 10 \text{ N}$, and $mg = 6.0 \text{ N}$
- $a = (10 - 6.0)/0.60 = 6.7 \text{ m/s}^2$ up
- Salt water: $F_b = 12 \text{ N}$, so $a = (12 - 6.0)/0.60 = 10 \text{ m/s}^2$ up

A 20% denser liquid gave 50% more acceleration, so a is not proportional to ρ

Quick check

A 2.0 kg block of volume $1.5 \times 10^{-3} \text{ m}^3$ is released under water. Find F_b and the block's acceleration.

Answer: $F_b = 15 \text{ N}$ and $mg = 20 \text{ N}$, so $a = (15 - 20)/2.0 = -2.5 \text{ m/s}^2$. The block sinks, speeding up at 2.5 m/s^2 .

Exam trap

Leaving out the weight. This was the most-missed point on a recent buoyancy question. The acceleration comes from the net force, so write that the weight mg is the same in both liquids and then compare $F_b - mg$, not F_b alone.

Thinking a denser liquid holds the block back. A denser liquid pushes up harder, so a block rising through it speeds up faster: in the example, 10 m/s^2 in salt water against 6.7 m/s^2 in fresh water.

Putting the object's density into $\rho V g$. That ρ is the fluid's. The object's own density matters only through its mass m .

Starting from another equation when the question says to start from Newton's second law. Write $\Sigma F = ma$ first, as asked, then put in the forces.

Quick check

A small rigid ball is held 2.0 m below the surface of a lake, then 10 m below. Compare the pressure on it and the buoyant force.

Answer: the pressure rises from 1.2×10^5 to $2.0 \times 10^5 \text{ Pa}$, but the buoyant force stays the same.

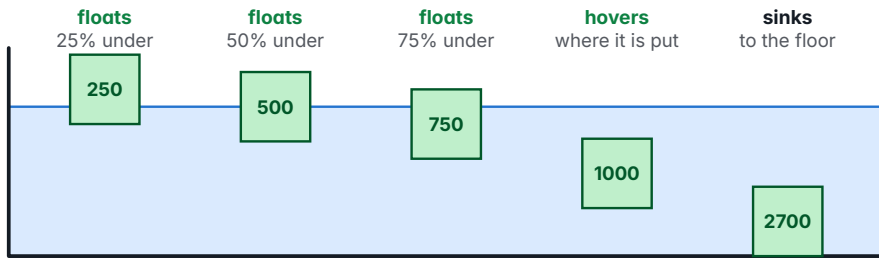
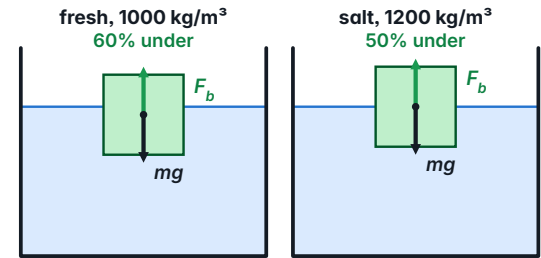
Quick check

Equal-volume aluminum and lead cubes hang from strings, fully under water. Which has the larger buoyant force? Which string has the larger tension?

Answer: the buoyant forces are equal, because the fluid and V are the same. The lead's string pulls harder: $T = mg - F_b$, and the lead has more mass.

8.3 Floating, weighing under water, and scales

A floating object displaces its own weight of fluid

Each number is the block's density in kg/m^3 ; the water's is 1000 kg/m^3 .The fraction under water is $\rho_{\text{object}} / \rho_{\text{fluid}}$.In denser water, less of the 600 kg/m^3 block has to be under to give the same $F_b = mg$.

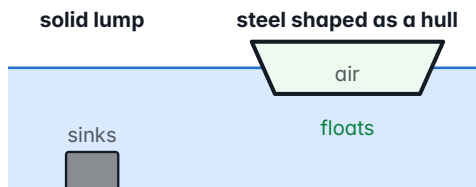
Floating at rest

With V_{sub} the volume under the surface and V the whole volume, $F_b = mg$ means $\rho_{\text{fluid}} V_{\text{sub}} g = \rho_{\text{obj}} V g$, so

$$\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{obj}}}{\rho_{\text{fluid}}}$$

not on the equation sheet

This holds only while it floats at rest with nothing else holding it. A block let go under water isn't floating yet: it first speeds up toward the surface.



A steel ship floats because its average density (the mass of steel and air, divided by the hull's outside volume) is less than water's.

Worked example

Take ice as 920 kg/m^3 and seawater as 1030 kg/m^3 . What fraction of a floating iceberg is under water?

$$1 \quad V_{\text{sub}}/V = 920/1030 = 0.893$$

About 89% is under water; only about a ninth shows

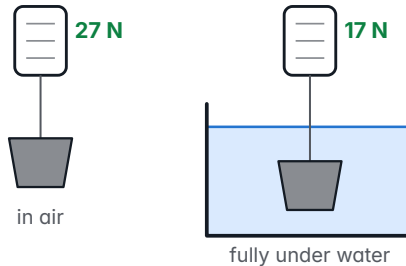
Worked example

A 60 kg raft has a volume of 0.10 m^3 . What is the largest load it can carry in fresh water without sinking?

- The buoyant force is largest with the raft fully under water: $F_b = 1000(0.10)(10) = 1000 \text{ N}$
- To float, the total weight must not exceed that: $(60 + m_{\text{load}})(10) = 1000$

$$m_{\text{load}} = 40 \text{ kg}$$

An object denser than the fluid sinks, and one less dense floats. If the two densities are equal, it stays wherever you put it.



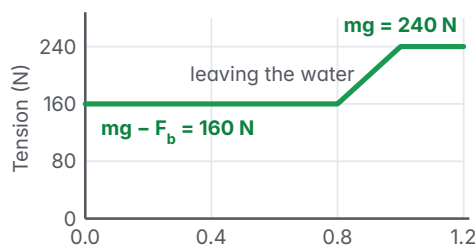
Under water the scale reads $T = mg - F_b$, so the drop in the reading is the buoyant force.

Worked example

A spring scale reads 27 N with a metal object hanging from it in air, and 17 N with the object fully under water. Find the object's volume and density.

- $F_b = 27 - 17 = 10 \text{ N}$
- $V = \frac{F_b}{\rho_w g} = \frac{10}{1000(10)} = 1.0 \times 10^{-3} \text{ m}^3$
- $m = 27/10 = 2.7 \text{ kg}$

$\rho = 2.7 \times 10^3 \text{ kg/m}^3$, found without measuring its size



Cube's bottom above the pool floor (m)

This graph shows the tension while a string pulls a cube slowly up out of a pool 1.0 m deep. While the cube is fully under, F_b and T stay constant; as it comes out, F_b falls and T rises to mg .

Quick check

From the graph alone, find the cube's mass, volume, density and side length.

Answer: $m = 240/10 = 24 \text{ kg}$. $F_b = 240 - 160 = 80 \text{ N}$, so $V = 80/(1000 \cdot 10) = 8.0 \times 10^{-3} \text{ m}^3$ and $\rho = 3.0 \times 10^3 \text{ kg/m}^3$. The tension rises between 0.8 m and 1.0 m, so the side is 0.20 m.

Exam trap

Thinking salt water pushes harder on a floating object. Anything floating at rest has $F_b = mg$, in any liquid. In salt water, less of the object has to be under the surface: the 600 kg/m^3 block above is 60% under in fresh water and 50% in salt water, with the same buoyant force. When the object is fully submerged, though, F_b is larger in the denser liquid.

Using the same equation for every string. A string from a scale pulls up, so $T + F_b = mg$. A string tied to the bottom of a tank pulls down, so $F_b = mg + T$. Draw the free-body diagram first and write the equation from it.

Using the whole volume of a floating object in $\rho V g$. Only the part under the surface pushes water aside. A 0.010 m^3 log floating with half its volume under water has $F_b = 1000(0.0050)(10) = 50 \text{ N}$, not 100 N.

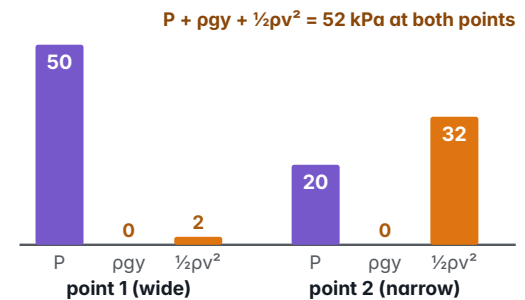
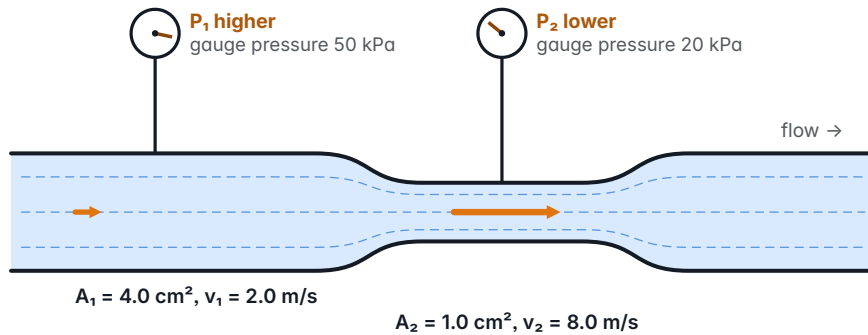
Quick check

A 0.50 kg ball floats with 40% of its volume under fresh water. Find its density and F_b . Then a string holds it fully under water. Find the tension.

Answer: $\rho = 0.40(1000) = 400 \text{ kg/m}^3$ and $F_b = mg = 5.0 \text{ N}$. Fully under, $V = 0.50/400 = 1.25 \times 10^{-3} \text{ m}^3$, so $F_b = 12.5 \text{ N}$ and $T = 12.5 - 5.0 = 7.5 \text{ N}$.

8.4 Continuity and Bernoulli's equation

A moving fluid conserves its mass and its energy



The bars are energy per cubic meter, in kPa (1 kPa = 1 kJ/m³). In the narrow part the kinetic term gains 30 kPa and the pressure loses the same 30 kPa.

Flow rate is volume per second

area A

length $v\Delta t$

$\Delta V = A \cdot v \Delta t$

$\Delta V / \Delta t = Av$

$Q = \frac{\Delta V}{\Delta t} = Av$

(in m³/s)

not on the equation sheet

The cylinder is the water that passes in a time Δt . The mass per second is ρAv , in kg/s.

$$A_1 v_1 = A_2 v_2$$

on the equation sheet

Whatever goes into a full pipe each second must come out each second, because an ideal fluid can't be squeezed into less space. So the water moves faster where the pipe is narrower.

Radius (or diameter)	Area	Speed
$\times 2$	$\times 4$	$\times \frac{1}{4}$
$\times \frac{1}{2}$	$\times \frac{1}{4}$	$\times 4$

When a pipe splits in two, the flow in equals the total flow out: $A_1 v_1 = A_2 v_2 + A_3 v_3$.

Worked example

A pipe of radius 2.0 cm fills a 10 L bucket in 8.0 s. Find the flow rate and the speed of the water in the pipe.

- 10 L = 0.010 m³, so the flow rate is $Q = 0.010/8.0 = 1.25 \times 10^{-3} \text{ m}^3/\text{s}$
 - $A = \pi(0.020)^2 = 1.26 \times 10^{-3} \text{ m}^2$
- $v = Q/A = 0.99 \text{ m/s}$, about 1.0 m/s

Why the pressure drops

Water entering the narrow part speeds up, so the water behind it must push harder than the water ahead. So at the same height, faster flow means lower pressure.

Bernoulli's equation on the equation sheet

For two points in the same steady flow:

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

P work the fluid behind does pushing each m³ along

ρgy gravitational energy per m³

$\frac{1}{2}\rho v^2$ kinetic energy per m³

Every term is in J/m³, the same unit as Pa, so this is conservation of energy per cubic meter.

How to use Bernoulli's equation

- Pick two points along the same steady flow, with no pump between them.
- Choose one height as zero, and use all absolute or all gauge pressures.
- At the **same height**, the pressure is lower where the water moves faster.
- At the **same speed** (same pipe area), the pressure is lower at the higher point: $P_1 - P_2 = \rho g(y_2 - y_1)$.

Worked example

Water at 2.0 m/s flows along a level pipe from $4.0 \times 10^{-4} \text{ m}^2$ into $1.0 \times 10^{-4} \text{ m}^2$. Find the new speed and the pressure difference.

- $v_2 = \frac{A_1 v_1}{A_2} = \frac{(4.0 \times 10^{-4})(2.0)}{1.0 \times 10^{-4}} = 8.0 \text{ m/s}$
- Same height: $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$
- $= \frac{1}{2}(1000)(64 - 4.0)$

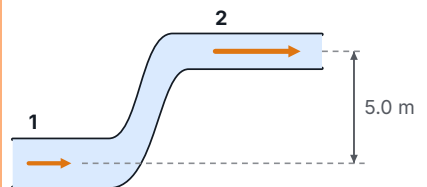
The pressure is $3.0 \times 10^4 \text{ Pa}$ higher in the wide part

Comparing the pressure at two points

Name Bernoulli's equation and what is the same at the two points, then compare: "A and B are at the same height and the water is faster at B, so its pressure is lower."

Worked example

Water flows at 2.0 m/s and a gauge pressure of $2.0 \times 10^5 \text{ Pa}$ in a 6.0 cm² pipe. The pipe rises 5.0 m to an upper floor, where its area is 3.0 cm². Find the speed and the gauge pressure upstairs.



- Continuity: $v_2 = 2.0 \times 6.0/3.0 = 4.0 \text{ m/s}$
- $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2)$
- $= 2.0 \times 10^5 - 6.0 \times 10^3 - 5.0 \times 10^4$

$v_2 = 4.0 \text{ m/s}$ and $P_2 = 1.44 \times 10^5 \text{ Pa}$ gauge

Exam trap

Confusing speed with flow rate. Along one pipe the flow rate (m³/s) is the same everywhere, but the speed (m/s) is not. In the level-pipe example, the flow rate stays $8.0 \times 10^{-4} \text{ m}^3/\text{s}$ while the speed goes from 2.0 to 8.0 m/s.

Using "faster means lower pressure" when the heights differ. The rule compares two points at the same height. In the rising-pipe example, 50 kPa of the 56 kPa drop comes from the 5.0 m climb, and only 6 kPa from the speed.

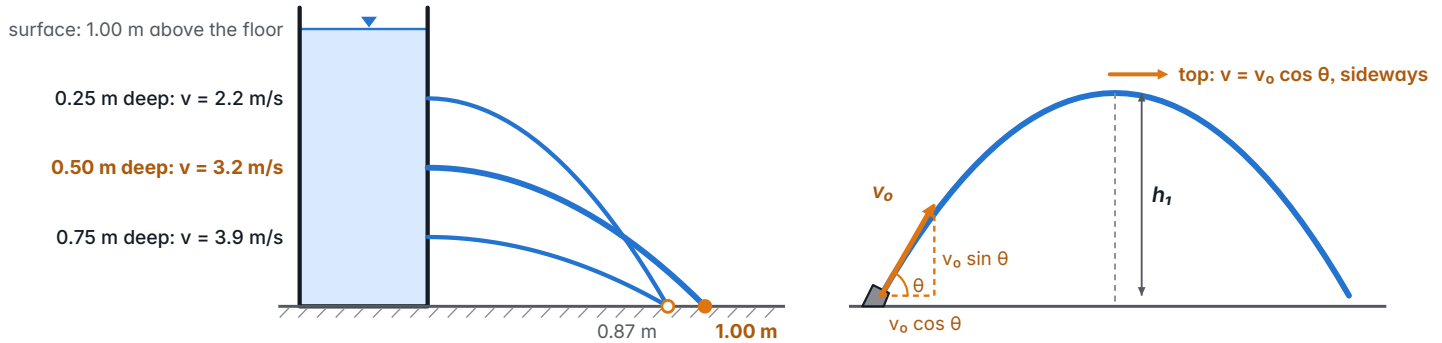
Quick check

A hose of inner radius 1.0 cm carries water at 1.5 m/s. Your thumb shrinks the opening to a radius of 0.50 cm, with the same flow rate. Find the exit speed and the flow rate.

Answer: a quarter of the area means 4 times the speed: 6.0 m/s. The flow rate is $\pi(0.010)^2(1.5) = 4.7 \times 10^{-4} \text{ m}^3/\text{s}$.

8.4 Jets, fountains and fluid labs

Water leaves a hole as fast as if it had fallen from the surface



A deeper hole gives a faster jet but less height to fall, so the top and bottom jets land at the same spot, 0.87 m out, and the middle one (orange label) reaches farthest, 1.00 m out.

The speed of water leaving a hole

Use Bernoulli's equation from the surface (1) to the jet (2), with $y = 0$ at the hole:

$$P_0 + \rho gh + 0 = P_0 + 0 + \frac{1}{2} \rho v^2$$

$$v = \sqrt{2gh} \quad \text{not on the equation sheet}$$

It is the speed of a stone dropped from the surface to the hole (Torricelli's result). Both points must be open to the air, so the P_0 terms cancel, and the tank must be wide, so its surface barely moves.

Worked example

A hole is 0.80 m below the surface of a wide open tank and 1.25 m above the floor. How far from the tank does the water land?

- $v = \sqrt{2(10)(0.80)} = 4.0$ m/s, moving horizontally
- Fall time: $t = \sqrt{2(1.25)/10} = 0.50$ s
- Distance = $vt = 4.0(0.50)$

2.0 m from the tank

Exam trap

Mixing up the two heights. The depth h of the hole below the surface sets the exit speed; the height H of the hole above the floor sets the fall time. In the example, $h = 0.80$ m gives 4.0 m/s and $H = 1.25$ m gives 0.50 s.

Setting the whole velocity to zero at a fountain's top. Only the vertical part is zero; the water still moves sideways. Write $0 = (v_0 \sin \theta)^2 - 2gh_1$, not $0 = v_0^2 - 2gh_1$.

Which hole sends water farthest?

$$x = vt = \sqrt{2gh} \sqrt{2H/g} = 2\sqrt{hH}$$

In the tank above, $h + H = 1.00$ m. The product hH is largest when $h = H = 0.50$ m, so that jet lands farthest, 1.00 m out; the holes at 0.25 m and 0.75 m tie at 0.87 m.

Deriving a fountain's launch speed and flow rate

- At the top, a height h_1 above the nozzle, the vertical velocity is zero, so $0 = (v_0 \sin \theta)^2 - 2gh_1$.
- So $v_0 = \frac{\sqrt{2gh_1}}{\sin \theta}$. Energy conservation gives the same result.
- For a nozzle of radius r_0 , the flow rate is $Q = Av_0 = \pi r_0^2 \frac{\sqrt{2gh_1}}{\sin \theta}$.

Use the whole launch speed in $Q = Av$, not $v_0 \sin \theta$: the water crosses the nozzle's opening at v_0 . Before you take $\sin \theta$, check your calculator is in degree mode (page 8).

Does a smaller nozzle send the water higher?

An answer that earns both points:

Yes, the new height h_2 is greater than h_1 . With the same flow rate, a smaller opening means the water leaves **faster**, because $Q = Av$. A faster launch at the same angle has a larger vertical velocity, so the water **rises higher**.

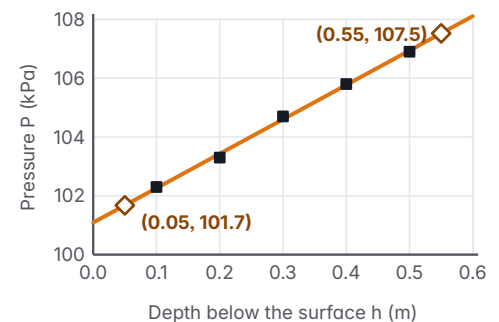
Worked example

Geneva's lakeside fountain sends about 0.50 m^3 of water a second straight up at about 200 km/h. How wide is the nozzle, and how high could the water go?

- $v = 200/3.6 = 55.6$ m/s (1 m/s = 3.6 km/h)
- $A = Q/v = 0.50/55.6 = 9.0 \times 10^{-3} \text{ m}^2$, so $r = \sqrt{A/\pi} = 5.4$ cm
- $h = v^2/2g = 55.6^2/20 = 155$ m

About 11 cm across, and 155 m high without air resistance

The real jet reaches only about 140 m, because air resistance slows it and it breaks into drops.



A pressure sensor took readings at five depths. Squares are the measurements; the diamonds are two points read off the best-fit line, which starts near the air's 1.0×10^5 Pa.

Worked example

Find the liquid's density from the graph.

- Slope = $\frac{107.5 - 101.7}{0.55 - 0.05} = 11.6$ kPa/m
- The slope is ρg , so $\rho = 11,600/10 \text{ kg/m}^3$

$$\rho \approx 1.2 \times 10^3 \text{ kg/m}^3$$

You measure	Graph	Slope
P at depths h	P vs h	ρg
jet speed v , depth h	v^2 vs h	$2g$
F_b , submerged V	F_b vs V	ρg
mass and volume	m vs V	ρ

For example, jets from holes 0.05 m to 0.25 m deep came out at 1.0 to 2.2 m/s. The best-fit line of v^2 against h had a slope of 19.4 m/s^2 , so $g \approx 9.7 \text{ m/s}^2$.

Quick check

A hole is 0.45 m below the surface of a wide open tank. Find the exit speed, then the speed later, when the surface is only 0.20 m above the hole.

Answer: 3.0 m/s, then 2.0 m/s: as the tank drains, the jet slows and lands closer.

Free-response questions about fluids

What earns the point on common fluids questions

The question asks	What earns the point
Compare the accelerations of one object in two liquids, using all the forces 8.3	Name both forces. The weight is the same, since it is the same object; $F_b = \rho V g$ is larger in the denser liquid, since the object is fully submerged in both (same V). So the net upward force and the upward acceleration are larger there.
Starting with Newton's second law, derive an expression for a 8.3	Write $\Sigma F = ma$ first, then $\rho V g - mg = ma$ taking up as positive, then solve for a . Use only the symbols the question lists.
Is your expression consistent with your claim? 8.3	Point to the variable, say where it sits (in the numerator, the denominator, under a root, next to a minus sign) and which way it moves the result: "Yes: ρ is in the numerator of the positive term, so a increases as ρ increases." Say "proportional" only when it is exactly true.
Calculate the force on a window or hatch 8.2	Find the pressure with $P = P_0 + \rho gh$. If the same air pressure pushes on the other side, only ρgh makes a net force. Multiply the pressure difference by the area and give the answer in newtons.
Sketch the pressure as a function of depth 8.2	A straight line with slope ρg , starting at P_0 for absolute pressure or at 0 for gauge. If you plot height above the bottom instead of depth, the line slopes down.
Find a liquid's density without weighing it 8.2	Measure the pressure at five or more depths with a pressure sensor and a meter stick, graph P against h , and divide the slope by g .
Derive the speed of water leaving a hole 8.4	Use Bernoulli's equation, or energy conservation, from the surface to the hole. The equal air pressures cancel and the surface barely moves, so $v = \sqrt{2gh}$.
Derive the volume flow rate 8.4	Multiply the opening's area by the whole exit speed, not just its vertical part: $Q = Av = \pi r^2 v$, in m^3/s .
With a smaller nozzle and the same flow rate, compare the heights 8.4	Give both steps of the reasoning: a smaller A means a larger v , because $Q = Av$ stays the same; at the same angle, a faster launch means the water rises higher.
Rank the pressures at points A, B and C in a pipe 8.4	Use Bernoulli's equation along the flow. At the same height, the pressure is lower where the water moves faster; at the same speed, it is lower at the higher point.
Explain why an object floats or sinks 8.1 8.3	Compare its average density with the fluid's, or compare its weight with the buoyant force it would feel fully under. "It's lighter than water" earns nothing, since a heavy log floats.

Where Unit 8 uses earlier units

- Unit 1** Water leaving a hole or a nozzle is a projectile, and at a fountain's top only the vertical velocity is zero.
- Unit 2** The buoyant force is one more force in $\Sigma F = ma$, and an object and the water push on each other as a third-law pair.
- Unit 3** Bernoulli's equation is energy conservation per cubic meter, and $v = \sqrt{2gh}$ is the free-fall speed.

Questions usually give water's density as 1000 kg/m^3 , which is 1 kg per liter. The air's $1.0 \times 10^5 \text{ Pa}$ equals the pressure of about 10 m of water.

⚠ Small things that cost points

Leaving the calculator in radian mode.

Angles in these problems are given in degrees. In radian mode $\sin 30$ comes out -0.988 instead of 0.500 , and calculators and Desmos usually start in radians. Set degree mode before any trig.

Mixing up ρ , P and p . Write what each symbol means before you substitute, such as " ρ = density of the salt water".

Substituting grams or centimeters.

Convert to kg , m and m^3 first, and give every answer its unit: 54 g is 0.054 kg , and 20 cm^3 is $2.0 \times 10^{-5} \text{ m}^3$.

The four free-response questions

- Question 1, Mathematical Routines** (10 points): calculate and derive, such as a fountain's launch speed.
- Question 2, Translation Between Representations** (12 points): draw or read graphs and diagrams and link them to equations.
- Question 3, Experimental Design and Analysis** (10 points): plan a lab, graph the data and find ρ or g from a slope.
- Question 4, Qualitative/Quantitative Translation** (8 points): a claim in words, a derivation, and whether they agree.

What a lab answer needs

- Say what you change, what you measure, and with what.
- Use at least five values of what you change (five depths, say), and repeat each measurement.
- Label both axes with units and draw one straight best-fit line, not a line joining the dots.
- Take the slope from two points on your line, far apart, then give the answer with units.

Set degree mode

TI-84 Plus CE. `mode`, arrow down to the row Radian Degree, highlight DEGREE, press `enter`, then `2nd mode` to quit.

TI-Nspire CX. `on` for the home screen, then 5 Settings, 2 Document Settings. Set Angle to Degree, move to Make Default and press `enter`.

Desmos. Open the wrench (graph settings) and choose Degrees.

Enter powers of ten

TI-84 Plus CE. Type 6.67 , then `2nd` `(-)` `11`. The screen shows $6.67\text{E}-11$. Use `(-)` for a negative sign, never the subtraction key.

TI-Nspire CX. Type 6.67 `x` `10` `^` `(-)` `11`. Use `(-)` for a negative sign, never the subtraction key.

Desmos. Type 6.67×10^{-11} .

Symbols that look alike

ρ	density	kg/m^3
P	pressure (or power)	Pa (or W)
p	momentum	$\text{kg}\cdot\text{m/s}$